

# Neutron Stars, Relativity and Black Holes

**Neutron Stars**  
**Pulsars**  
**Neutron Star Binaries**  
**Gamma Ray Bursts**  
**Relativity**  
**Black Holes**

# Neutron Stars

After a **Type I supernova**, little or nothing remains of the original star.

After a **Type II supernova**, part of the core may survive. It is very dense – as dense as an atomic nucleus – and is called a **neutron star**.

# Neutron Stars

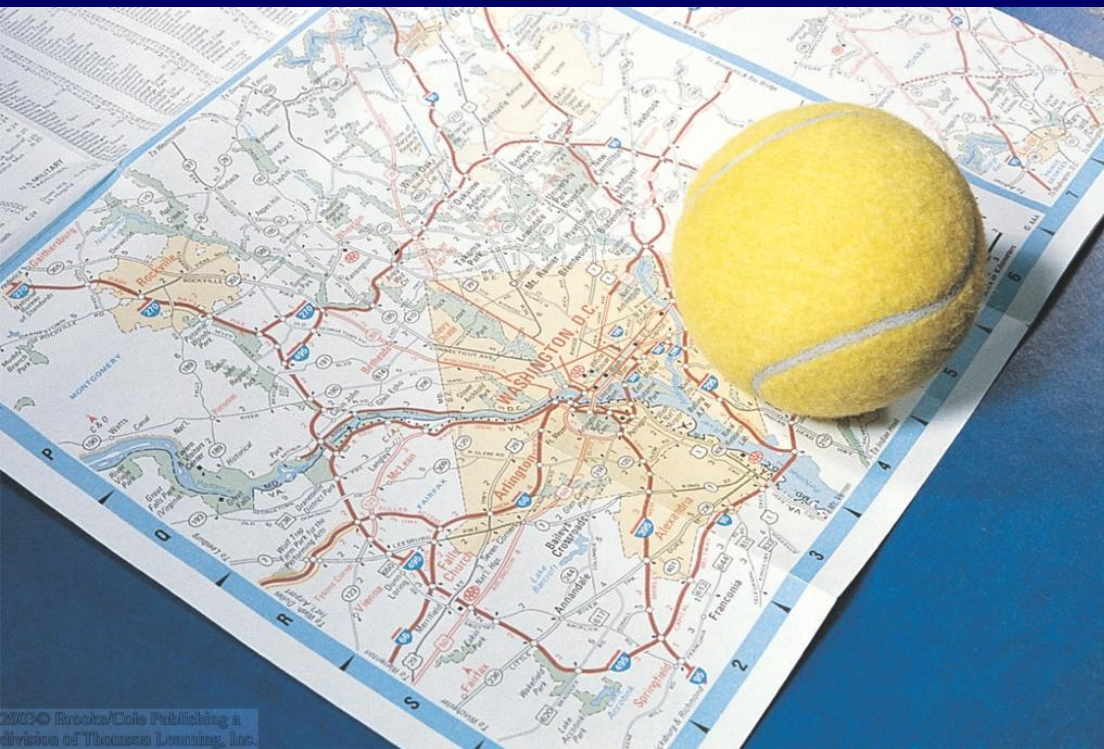
- A supernova explosion of an  $M > 8 M_{\odot}$  star blows away its outer layers.
- The central core will collapse into a compact object of  $\sim$ a few  $M_{\odot}$ .
- Pressure becomes so high that electrons and protons combine to form stable neutrons throughout the object.

Typical size:  $R \approx 10$  km

Mass:  $M \approx 1.4 - 3 M_{\odot}$

Density:  $\rho \approx 10^{14}$  g/cm<sup>3</sup>

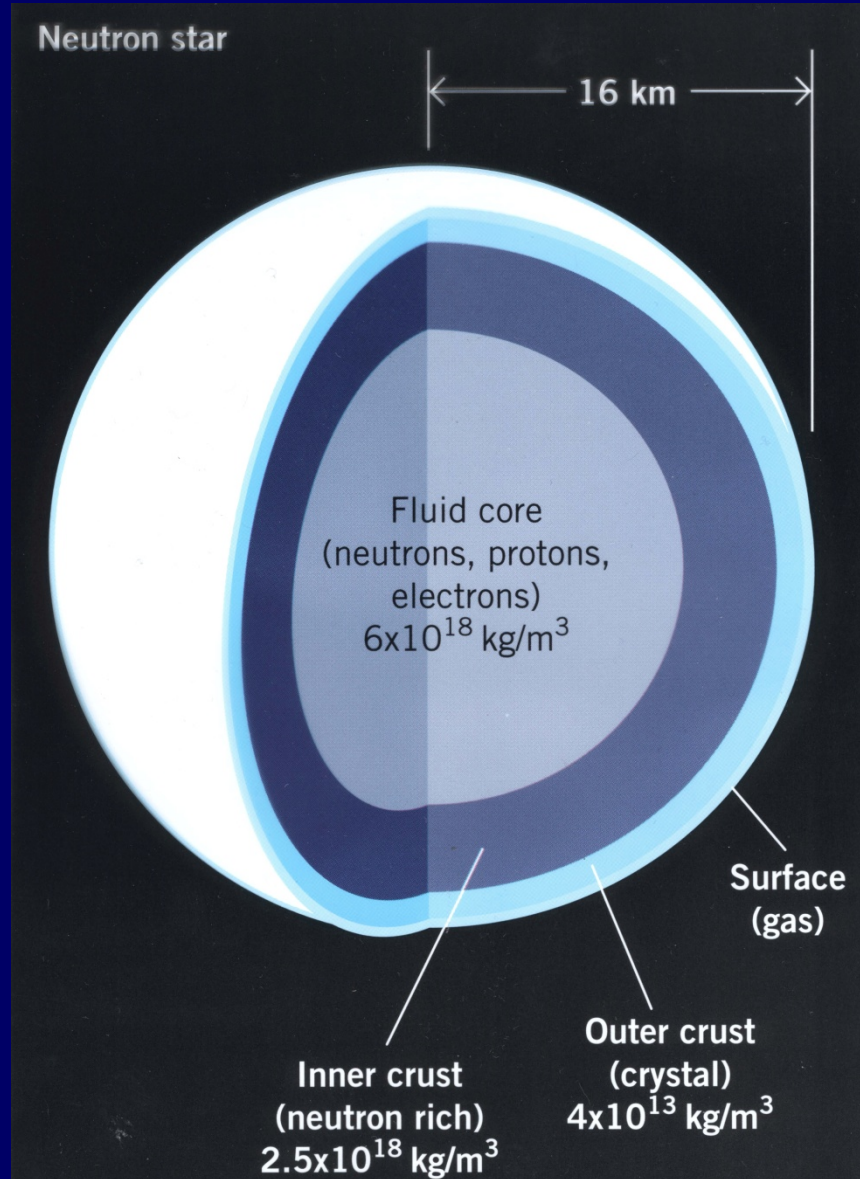
- A piece of a neutron star the size of a sugar cube has a mass of  $\sim$ 100 million tons!!!



# Neutron Stars

Other important properties of neutron stars (beyond mass and size):

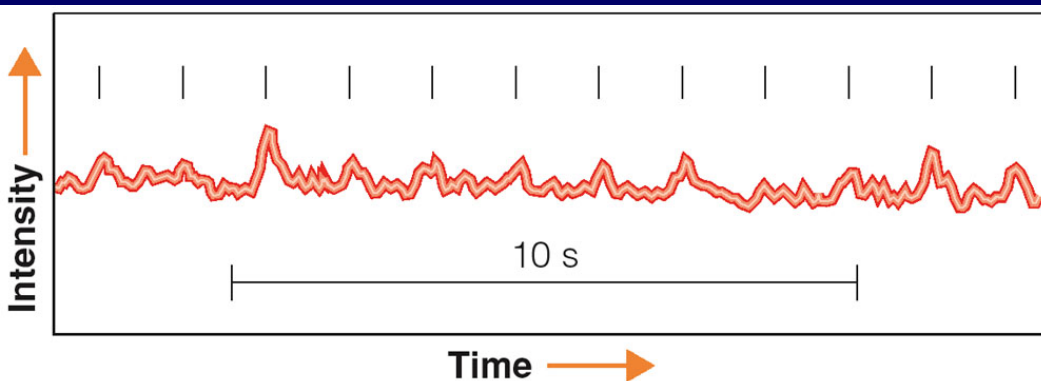
- **Rotation** – as the parent star collapses, the neutron core spins very rapidly, conserving angular momentum. Typical periods are fractions of a second.
- **Magnetic field** – again as a result of the collapse, the neutron star's magnetic field becomes enormously strong.



# Discovery of Pulsars (1967)



- Angular momentum must be conserved  
⇒ Collapsing stellar core spins up to periods of ~a few milliseconds.
- Magnetic fields are amplified up to  $B \approx 10^9 - 10^{15}$  G (up to  $10^{12}$  times the average magnetic field of the Sun)  
⇒ Rapidly pulsed (optical and radio) emission from some objects interpreted as spin period of neutron stars

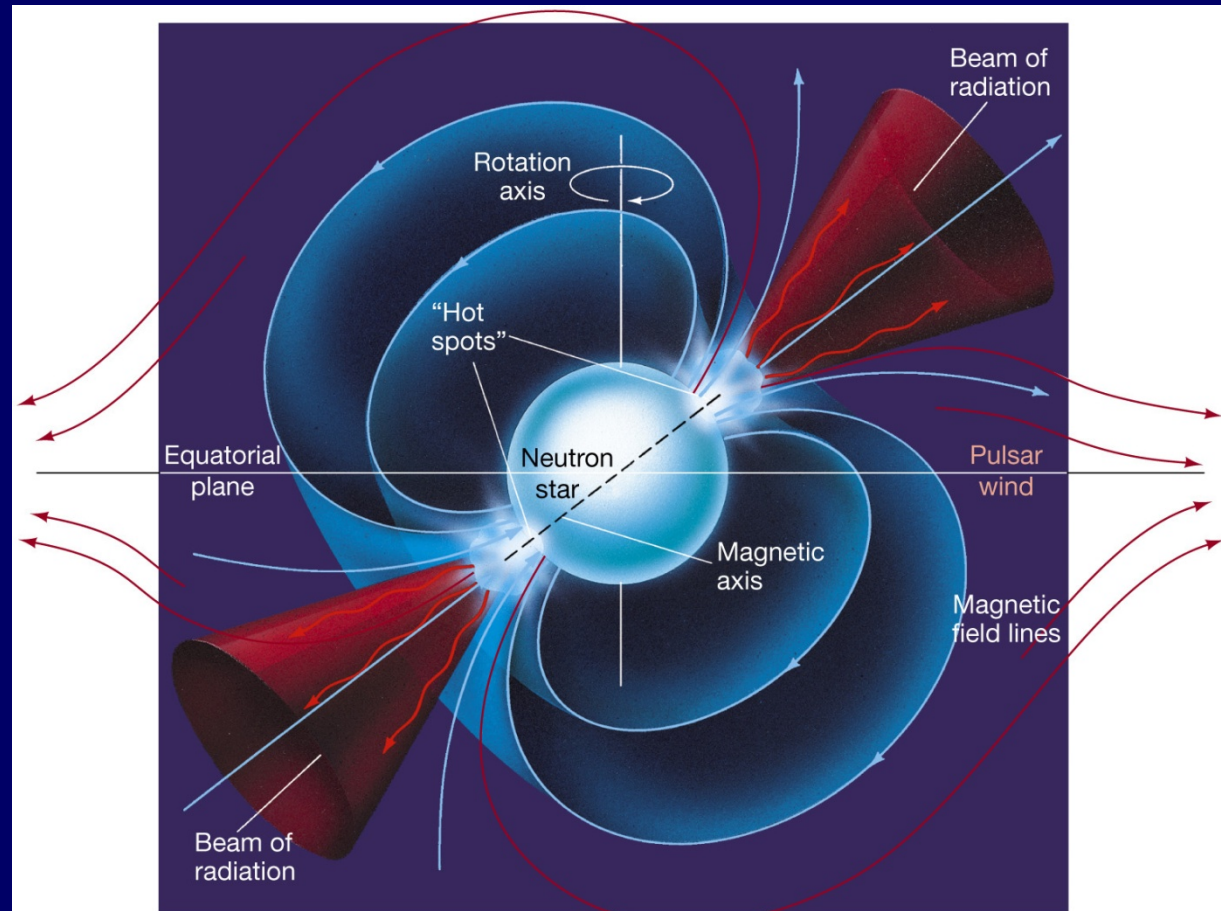


# Pulsars

But why would a neutron star flash on and off?

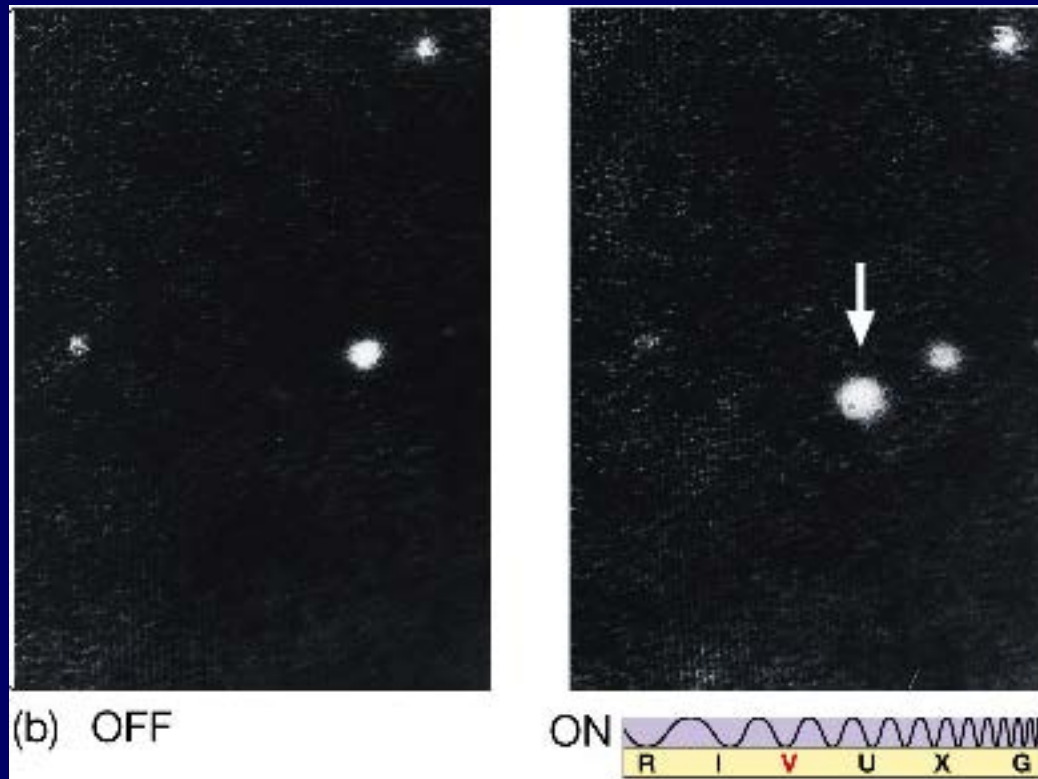
This figure illustrates the **lighthouse model** responsible

Strong **jets of matter** are emitted at the magnetic poles, as that is where they can escape. If the **rotation axis**  $\neq$  magnetic axis, the two beams will sweep out circular paths. If the Earth lies in one of the beam paths, we will see the star **blinking** on and off.

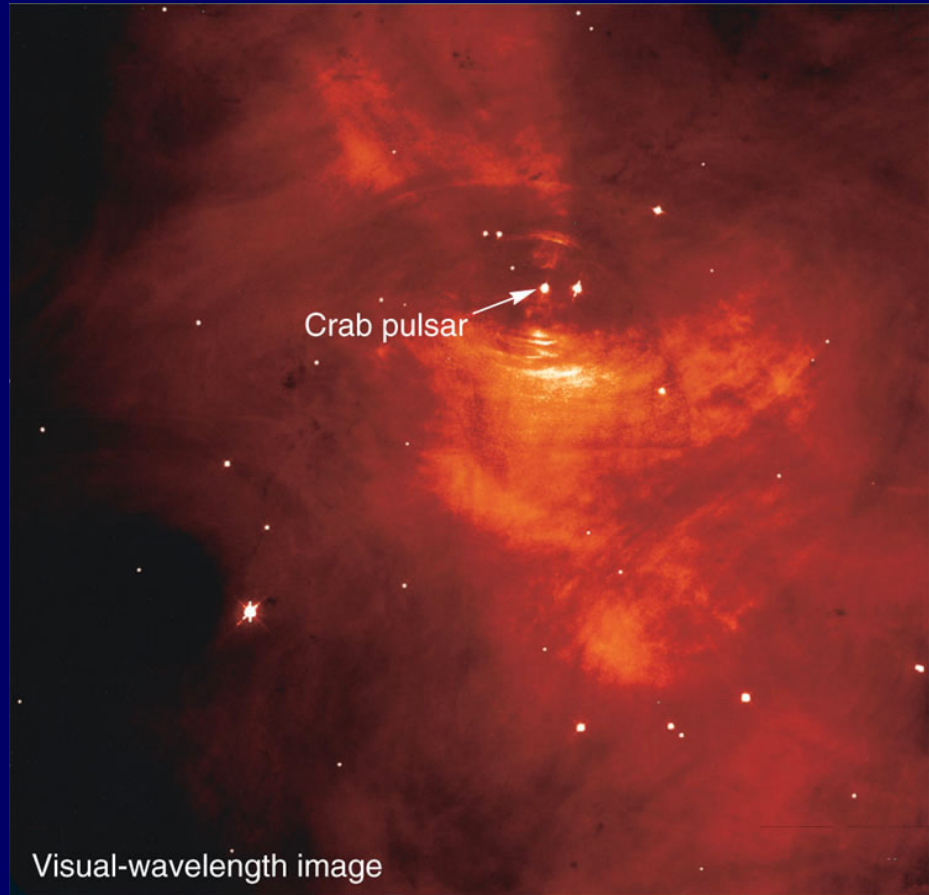


# Pulsars

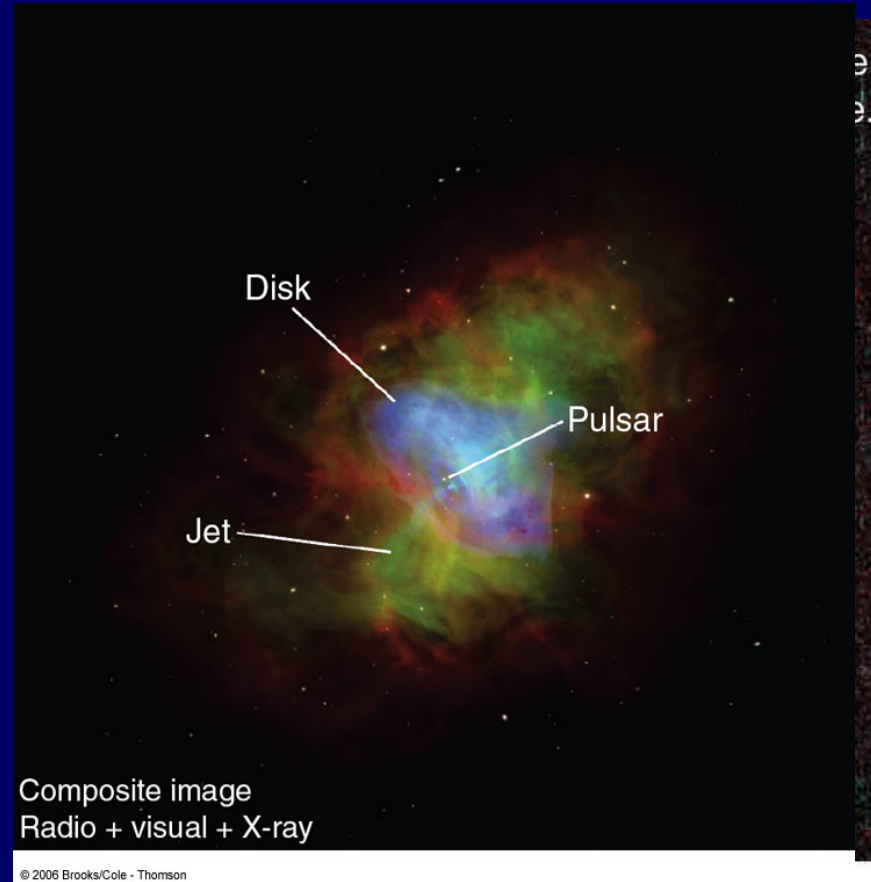
There is a pulsar at the center of the **Crab Nebula**. It is the only pulsar which pulses in the visible. Images below show it in the “off” and “on” positions. It also pulses in the gamma ray spectrum.



# Images of Pulsars and other Neutron Stars



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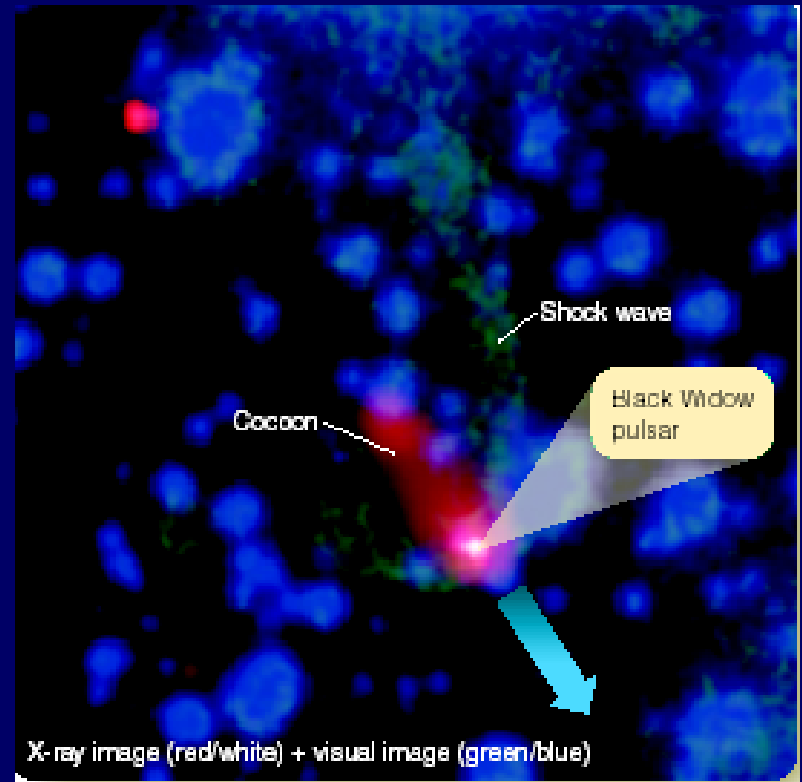


# Proper Motion of Neutron Stars

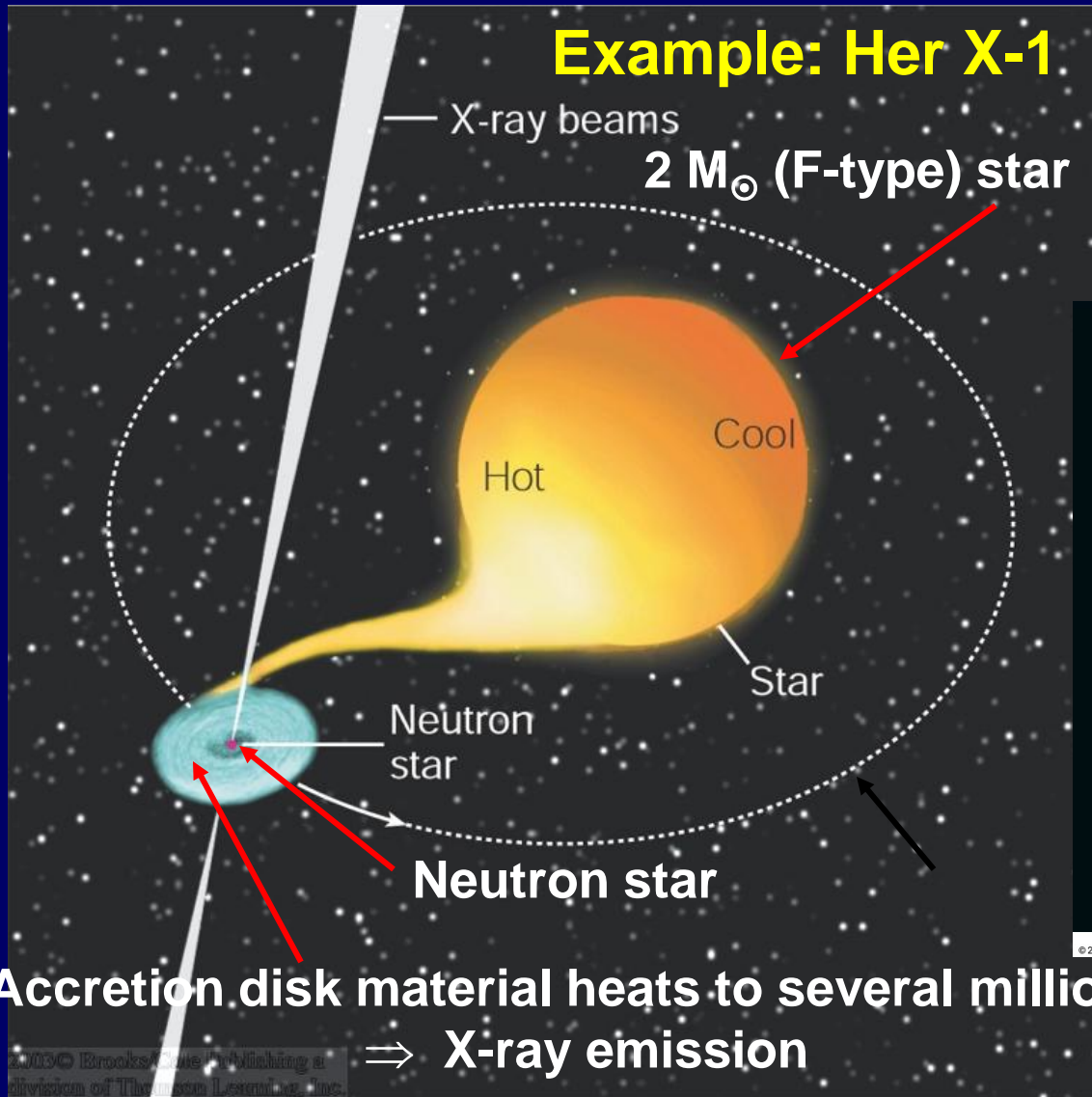


This might be a result of anisotropies during the supernova explosion forming the neutron star.

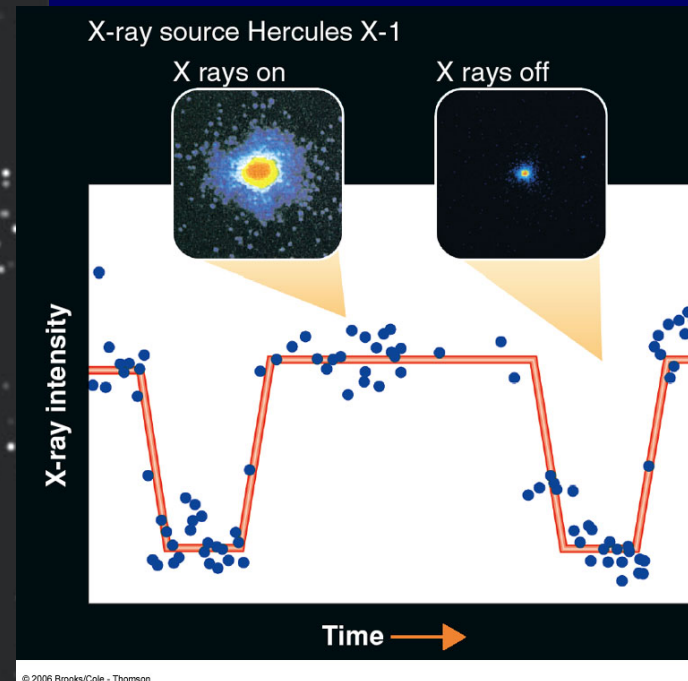
Some neutron stars are moving rapidly through interstellar space.



# Neutron Stars in Binary Systems: X-ray binaries



Star eclipses the neutron star and the accretion disk periodically



Orbital period  
= 1.7 days

# Jets of Energy from Compact Objects

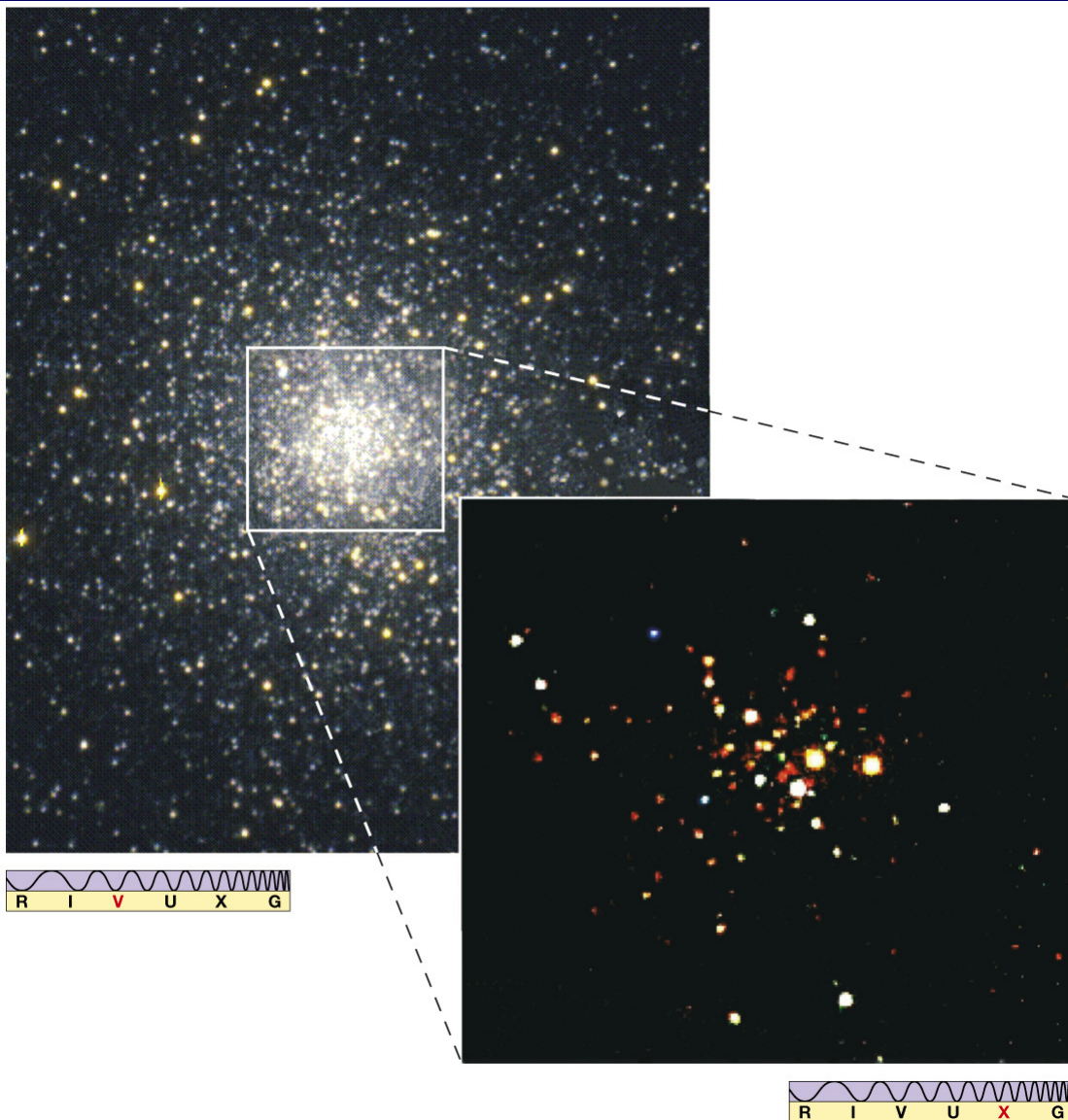


# Neutron-Star Binaries

Most pulsars have periods between 0.03 and 0.3

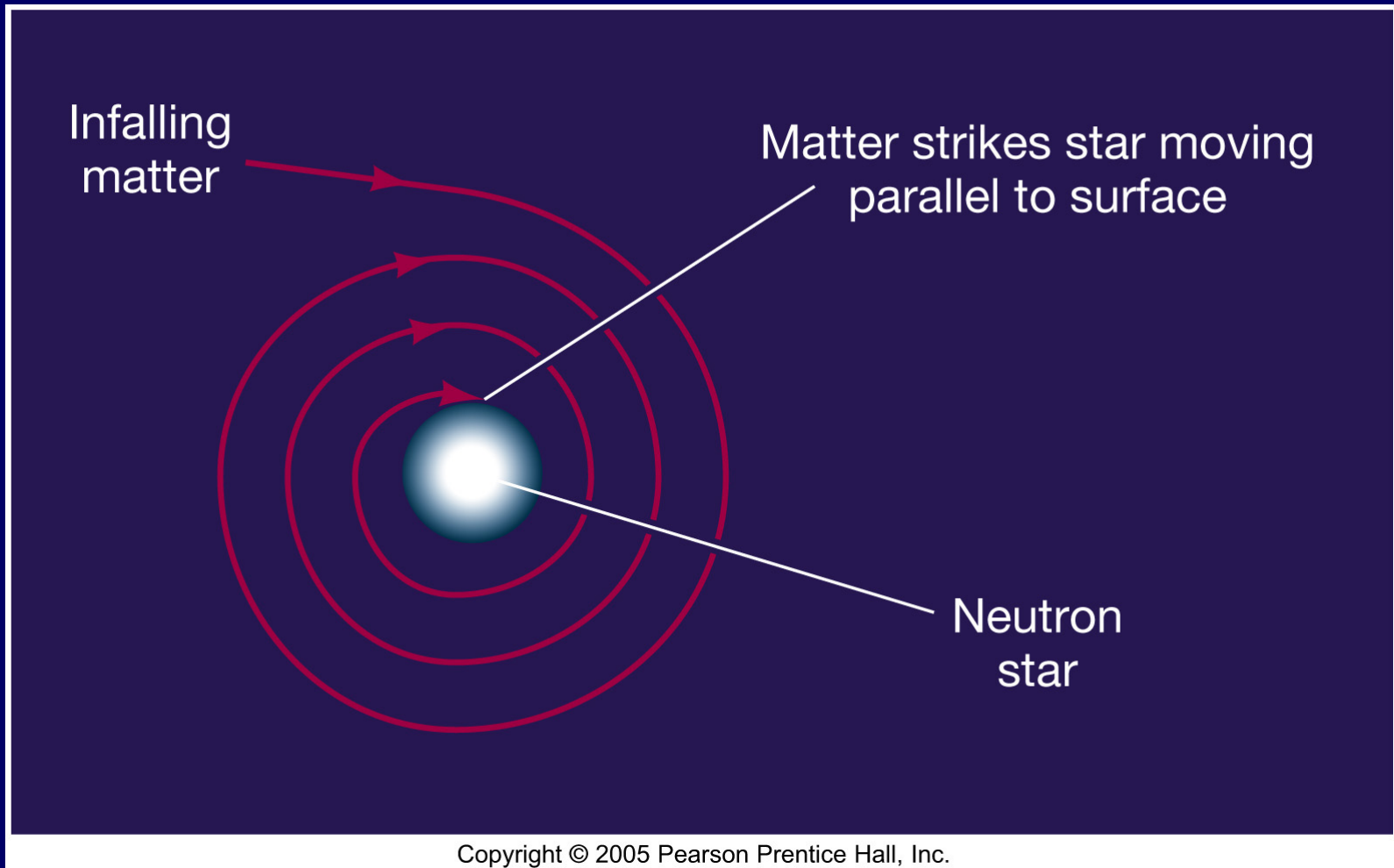
seconds, but a new class of pulsar was discovered in the early 1980s: the **millisecond pulsar**.

This globular cluster has been found to have 108 separate X-ray sources, about half of which are thought to be millisecond pulsars.



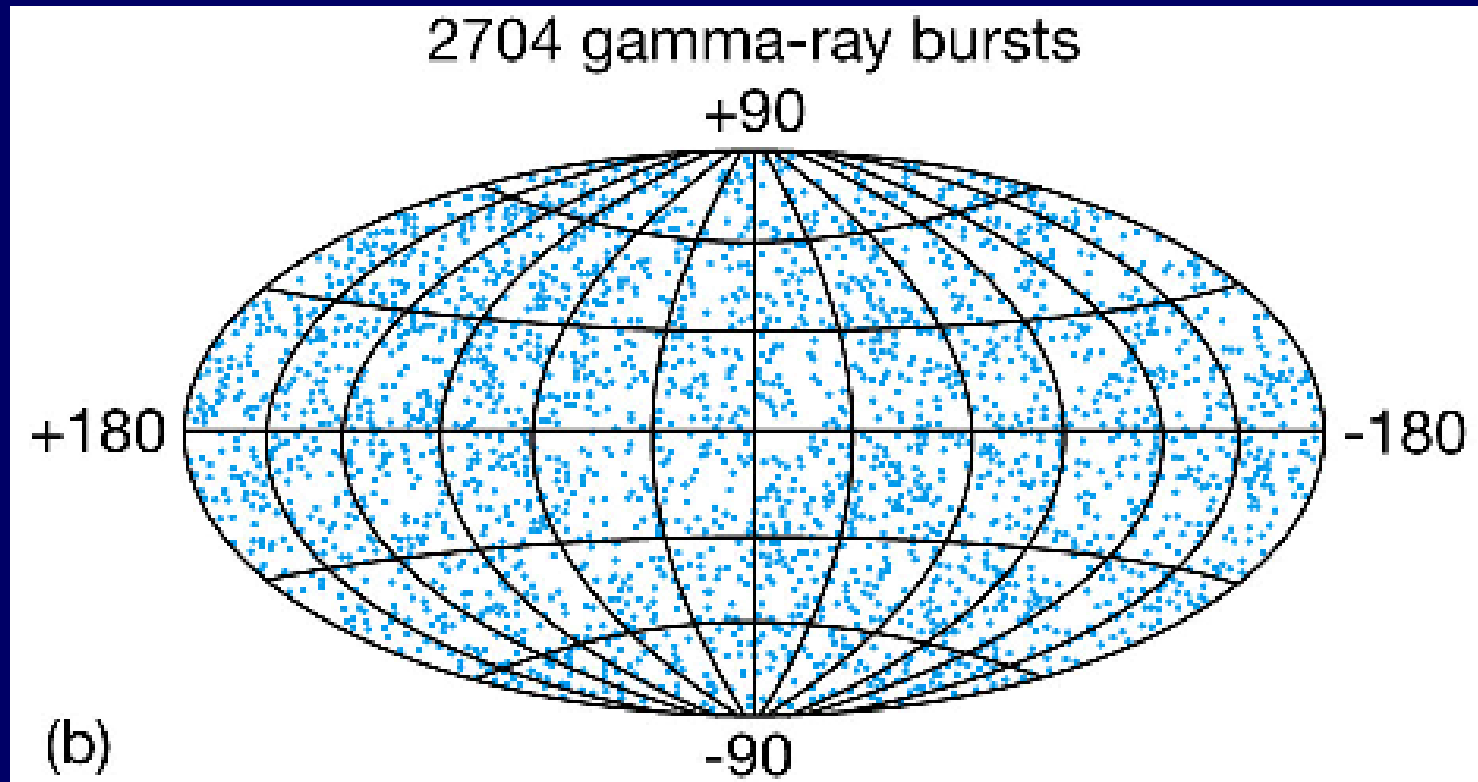
# Neutron-Star Binaries

Millisecond pulsars are thought to be “**spun-up**” by matter falling in from a companion.



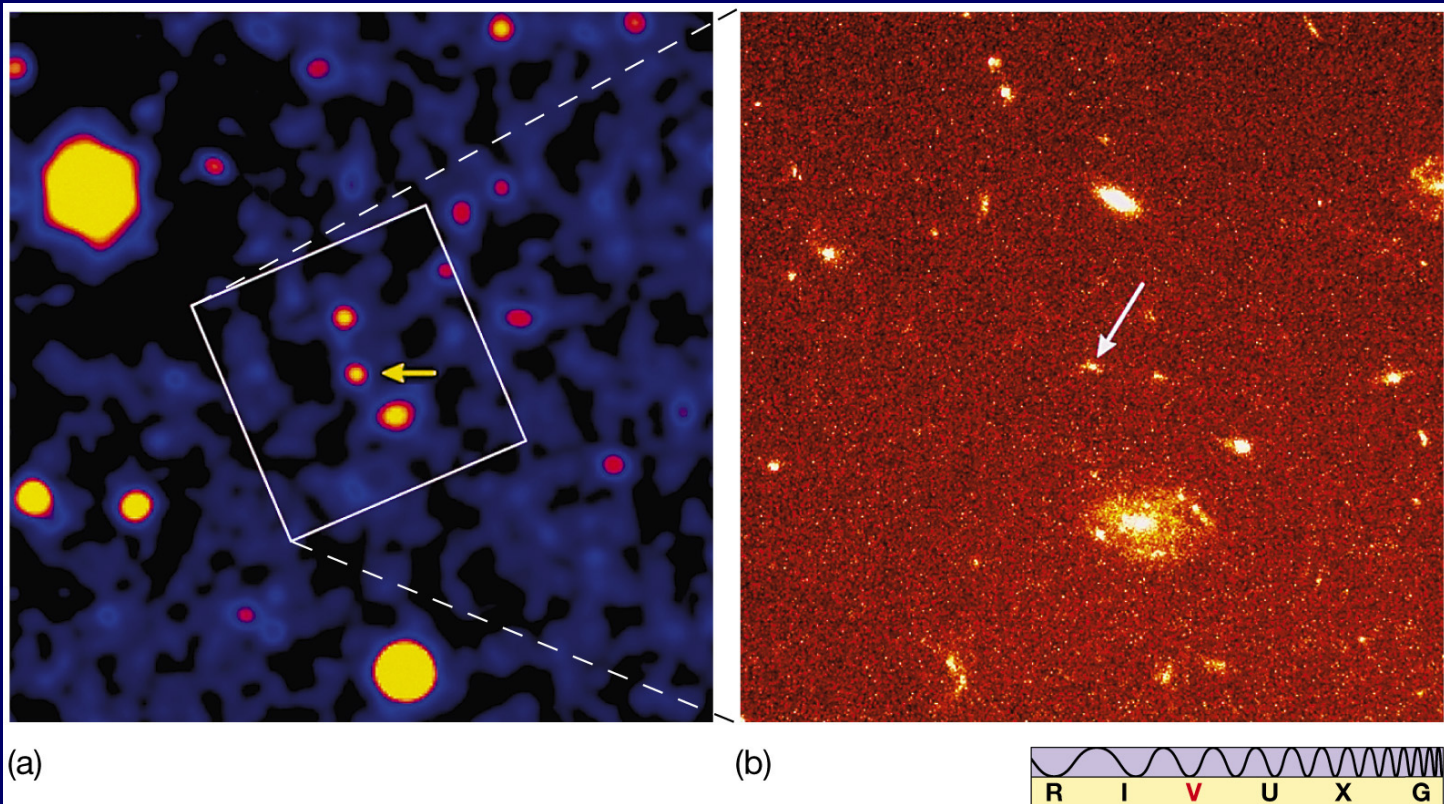
# Gamma-Ray Bursts

Gamma-ray bursts were first spotted by satellites looking for violations of nuclear test-ban treaties. This map (in **galactic coordinates**) shows where the bursts have been observed. There is no “clumping” of bursts anywhere, particularly not within the **Milky Way**. Therefore, the bursts must originate from **outside** our Galaxy.



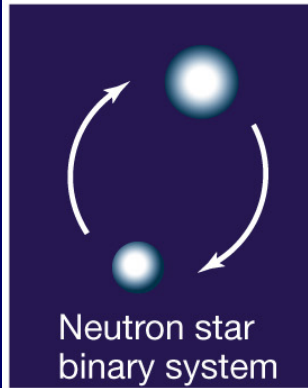
# Gamma-Ray Bursts

- Occasionally the spectrum of a burst can be measured, allowing distance determination.
- Distance measurements of some gamma bursts show them to be very far away – 2 billion parsecs for the first one measured.

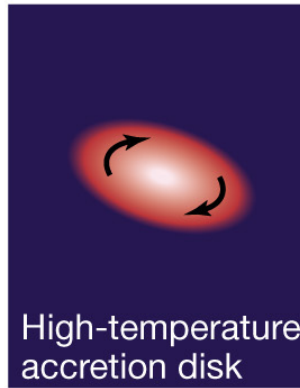
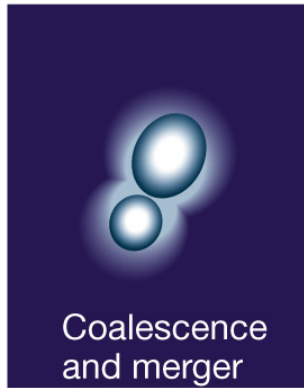


# Gamma-Ray Bursts

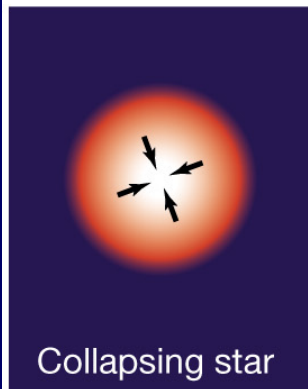
Two models – **merging neutron stars** or a **hypernova** – have been proposed as the source of gamma-ray bursts.



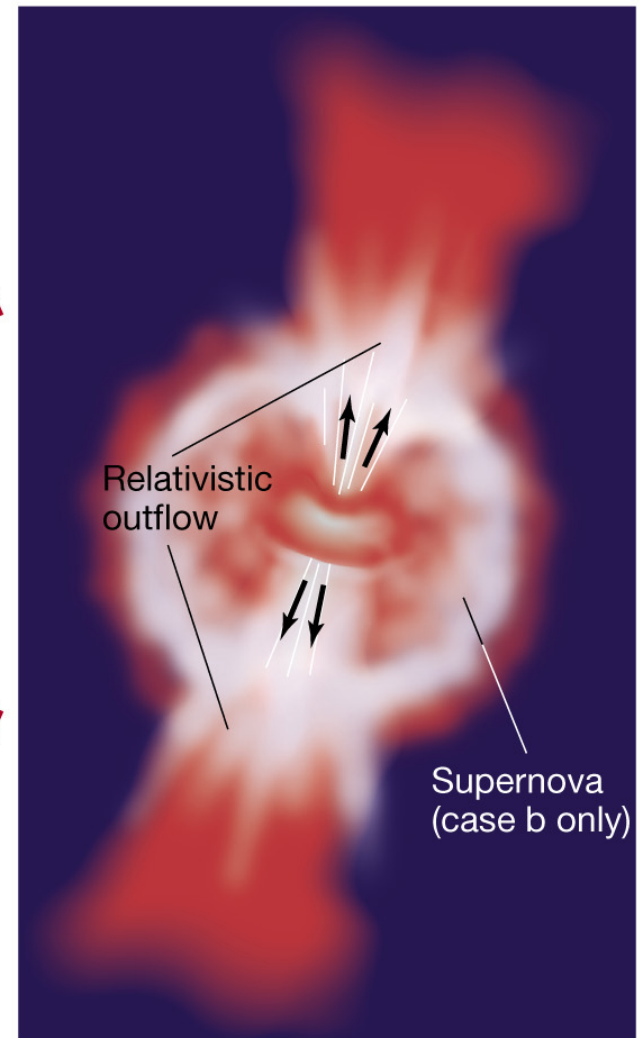
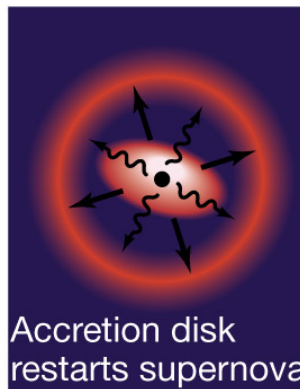
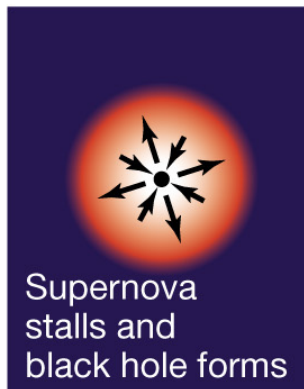
(a) Merging stars



Time →



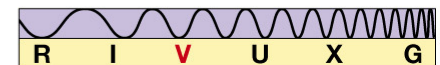
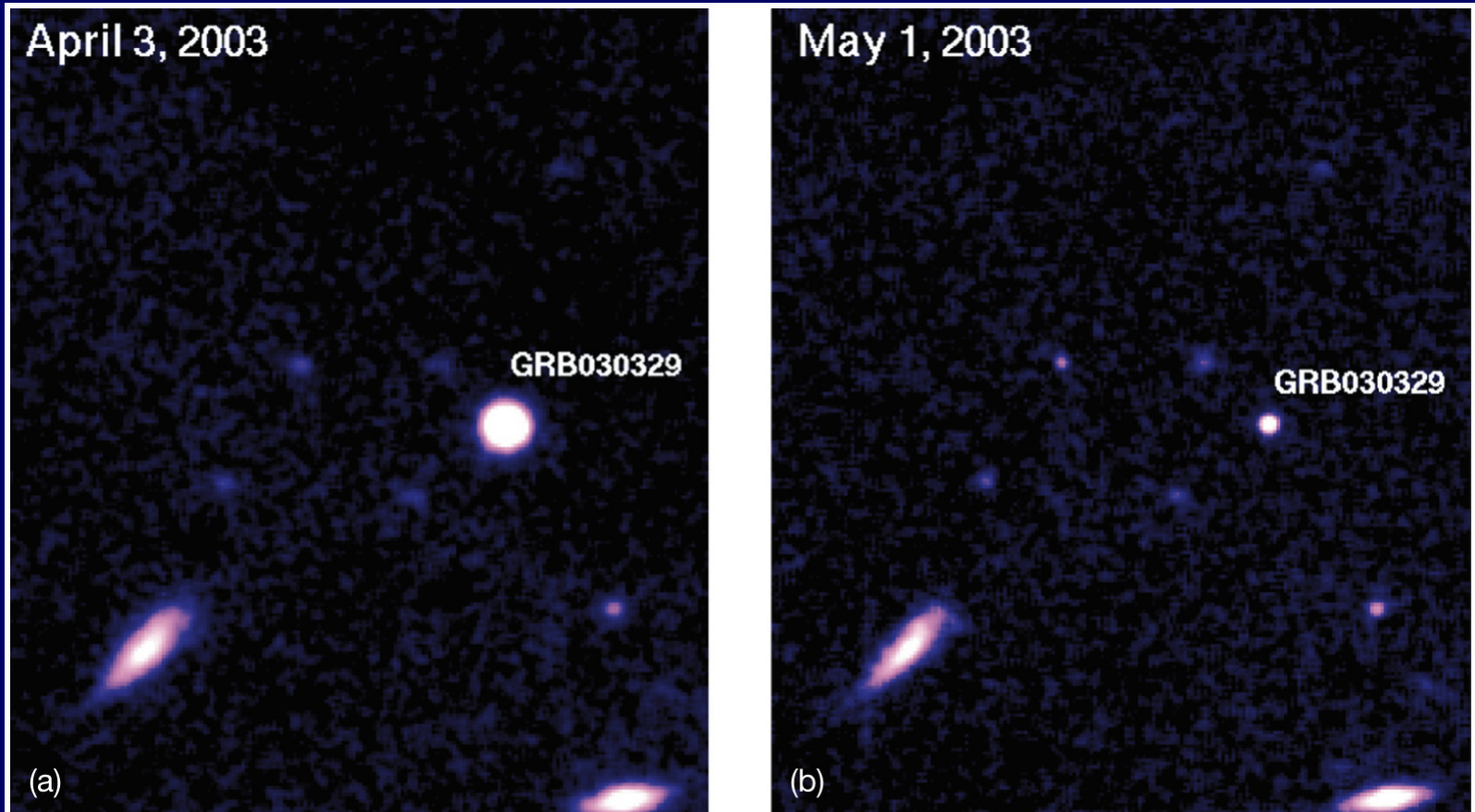
(b) Hypernova





# Gamma-Ray Bursts

This burst looks very much like an exceptionally strong supernova, lending credence to the **hypernova** model.



# Einstein's Theories of Relativity

- James Clerk Maxwell synthesized empirical formulas of electricity and magnetism into an electromagnetic theory. In 1865, he used his theory to show that light was an electromagnetic wave. In his theory, there was no provision for the speed of light to transform like mechanical speeds do in Newtonian theory.
- In 1887, Michelson and Morley did an experiment to measure the variation in the speed of light with respect to the direction of the Earth's motion around the Sun. They found no variation—light always traveled at the same speed.
- In 1904, Hendrik Lorentz derived transformation equations which kept the speed of light constant in inertial reference frames.
- In 1905, Einstein made the constancy of the speed of light in all inertial reference frames a postulate of his Special Theory of Relativity.

# Einstein's Theories of Relativity

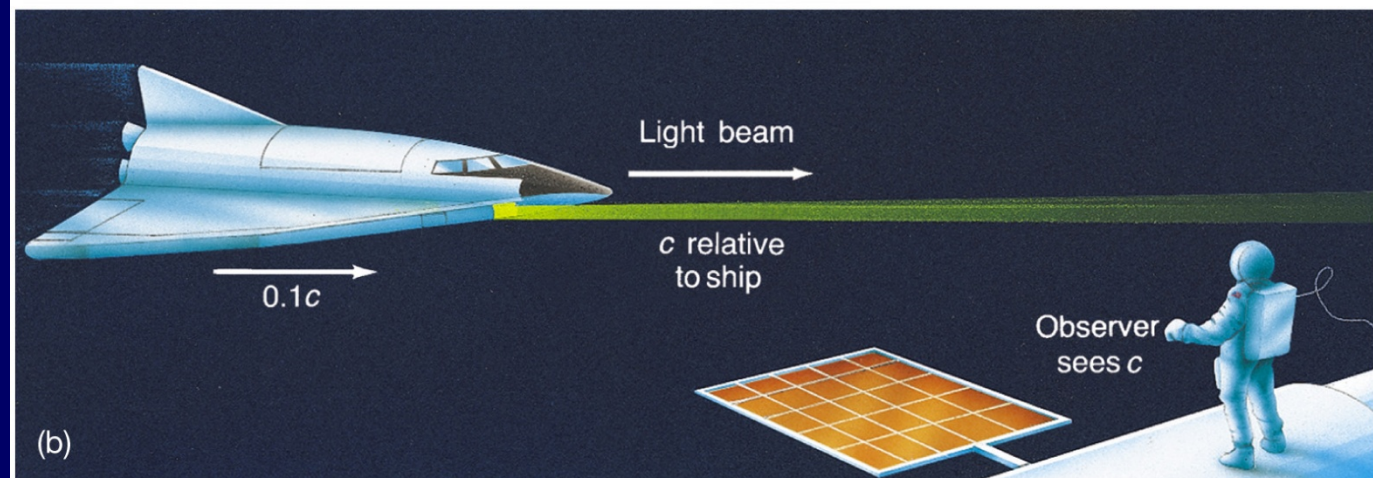
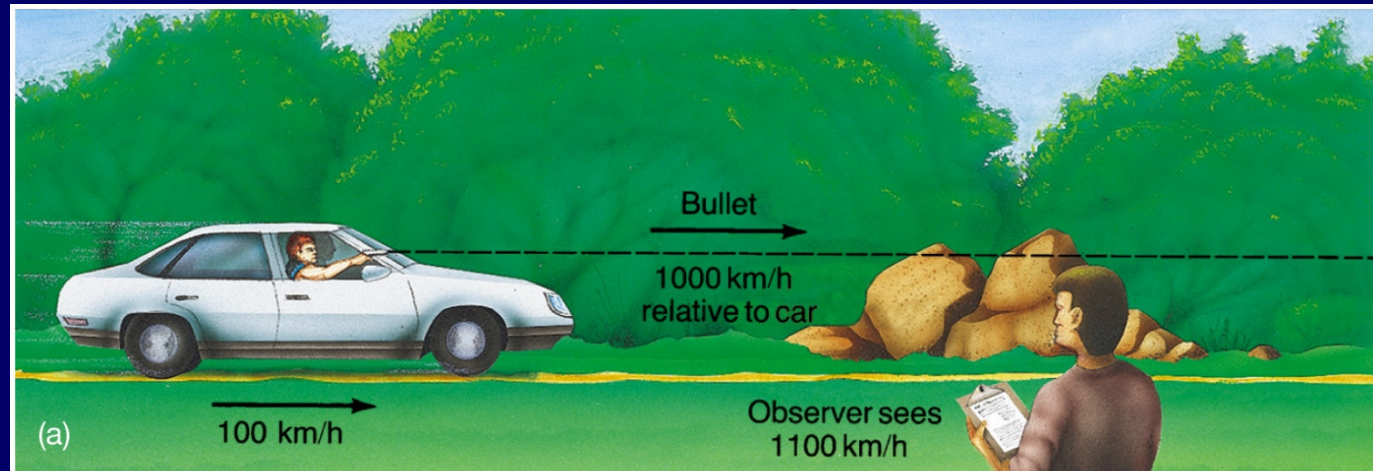
## Postulates of the Special Theory of Relativity:

1. The speed of light  $c$  is the same in all **inertial reference frames**. It is the maximum speed for transmitting energy and information.

2. The laws of physics are **Lorentz invariant** in any inertial reference frame.

Lorentz invariant means not changing under **Lorentz transformations**.

One must use 4-dimensional space-time reference frames to satisfy these postulates.



# Effects of Special Relativity

The postulates of the special theory of relativity imply results some of which are:

- Lengths will appear longer to a stationary observer for a system moving near the speed of light
- Time will appear slower to a stationary observer for a system moving near the speed of light
- Mass and energy are equivalent. This equivalency is expressed in Einstein's famous equation:

$$E = mc^2$$

# Einstein's Theories of Relativity

**General Relativity** extends **Special Relativity** to non-inertial reference frames. It assumes Special Relativity for inertial reference frames.

Non-inertial reference frames are those that **accelerate** with respect to each other.

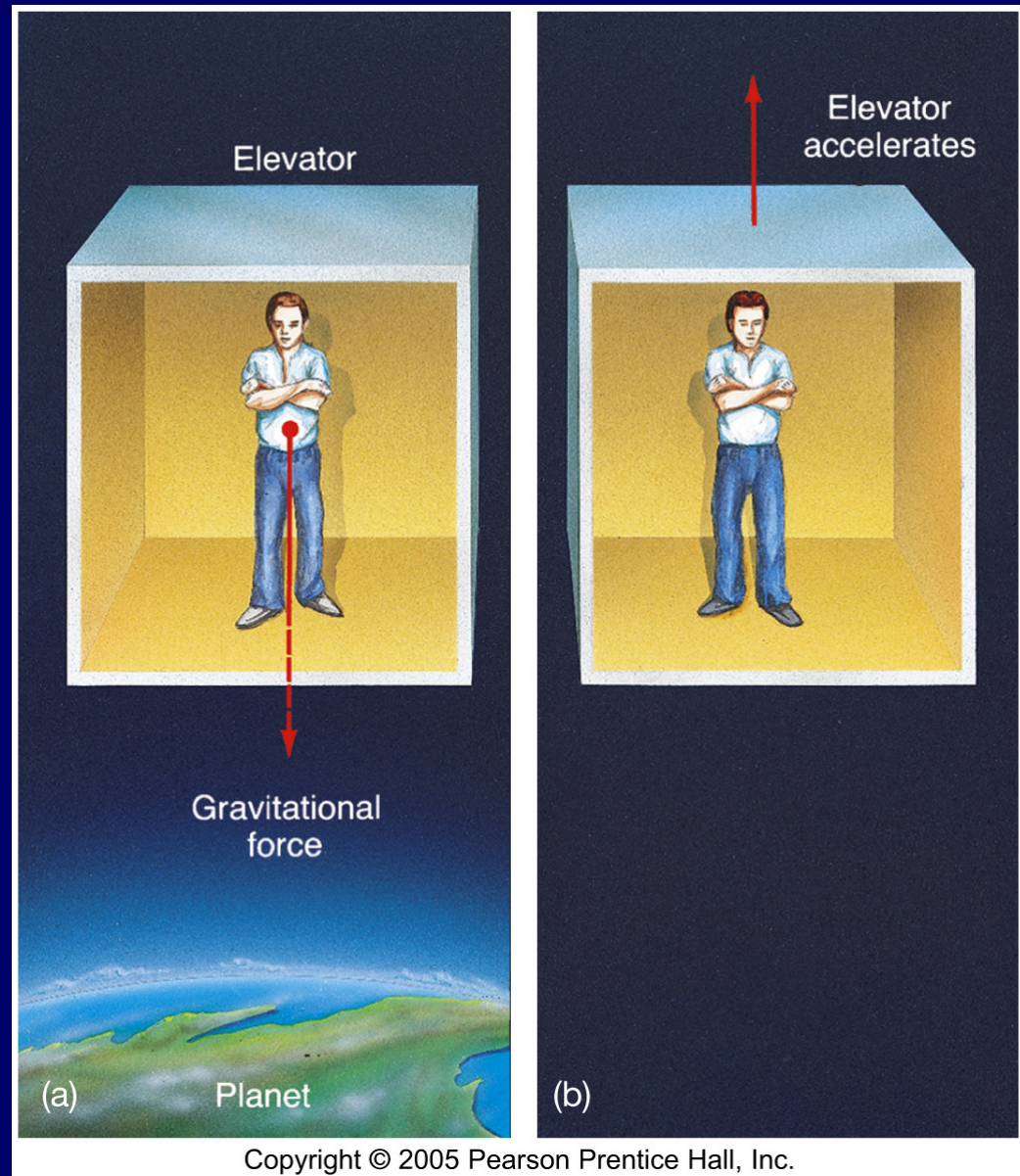
## Postulates of the General Theory of Relativity:

1. **Principle of Equivalence:** gravitational and inertial mass are equivalent.
2. The laws of physics are **generally covariant** in any frame of reference.

**General covariance** means that physical laws will have terms in them related to the acceleration that vanish in inertial reference frames.

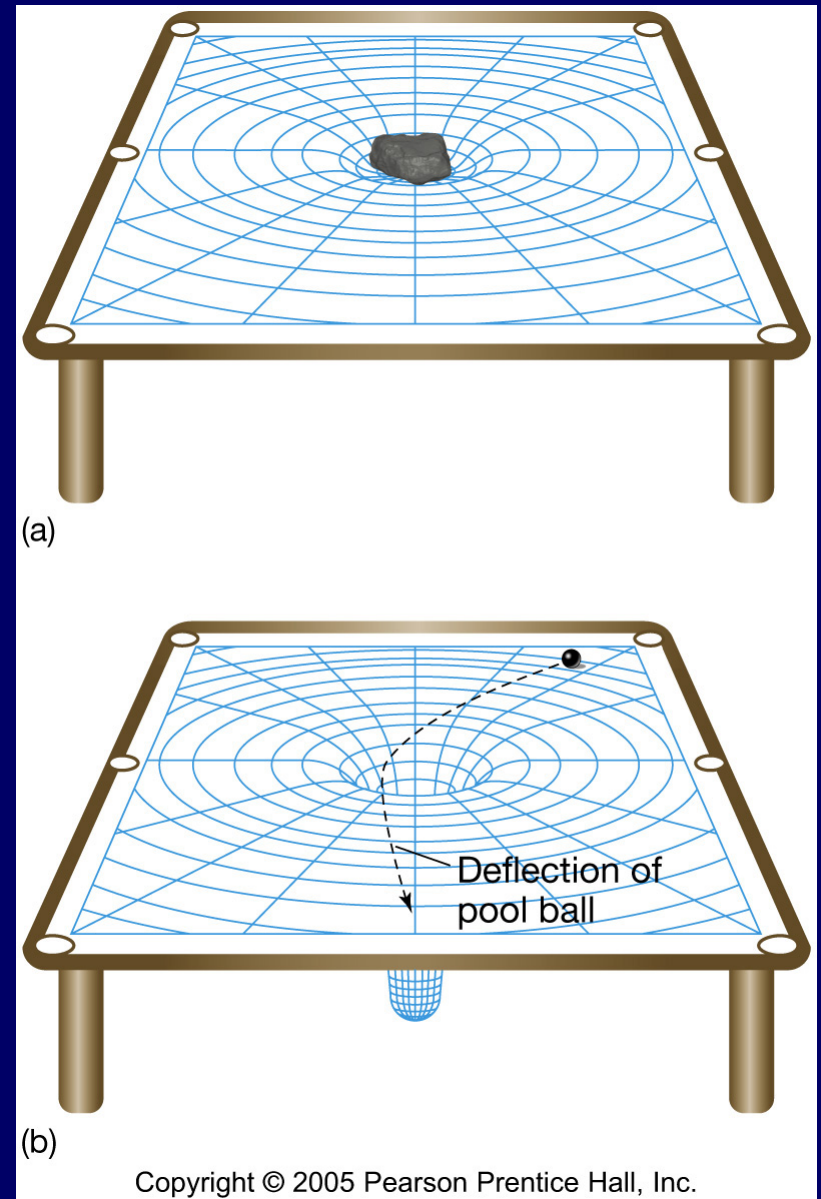
# Einstein's Theories of Relativity

- The **Principle of Equivalence** implies that it is impossible to tell, from within a closed system, whether one experiences a force from a gravitational field or from dynamic acceleration.
- Einstein showed that this hypothesis implies that light is bent by a large gravitational field because of the equivalency of matter and energy. The light beam has energy that is attracted by a gravitational field.



# Einstein's Theories of Relativity

- Bending of starlight means that matter appears to **warp** space-time and in doing so redefines straight lines (the path a light beam would take).
- The larger the mass, the deeper space-time is warped.
- The deflection of the ball is like the bending of a photon by a large mass.



# Black Holes

Just like white dwarfs, which have a Chandrasekhar limit:  $1.4 M_{\odot}$ , there is a mass limit for neutron stars.

**Neutron stars can not exist  
with masses  $> \sim 3 M_{\odot}$**

We know of no mechanism to halt the collapse of a compact object with  $> \sim 3 M_{\odot}$ .

It will collapse into a single point – a singularity:

⇒ A black hole!



# Escape Velocity

The velocity needed to escape Earth's gravity from the surface:

$$v_{\text{esc}} \approx 11.6 \text{ km/s.}$$

Gravitational force decreases with distance  
( $\sim 1/d^2$ )

$\therefore$  Starting out high above the surface

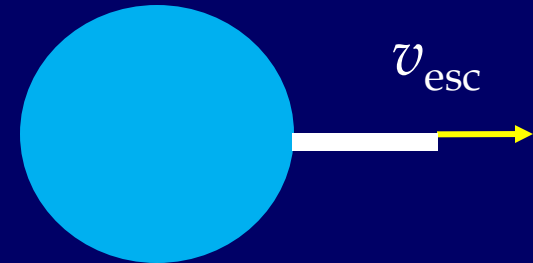
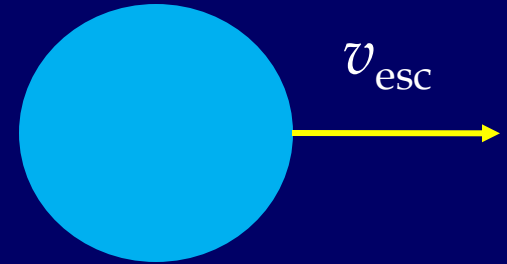
$\Rightarrow$  lower escape velocity.

If you could compress the Earth to a smaller radius

$\Rightarrow$  higher escape velocity from the surface.



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# The Schwarzschild Radius

There is a limiting radius where the escape velocity reaches the speed of light  $c$ :

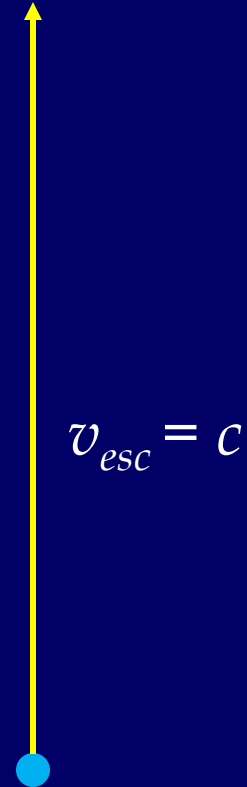
$$R_s = \frac{2GM}{c^2}$$

$G$  = gravitational constant

$M$  = mass

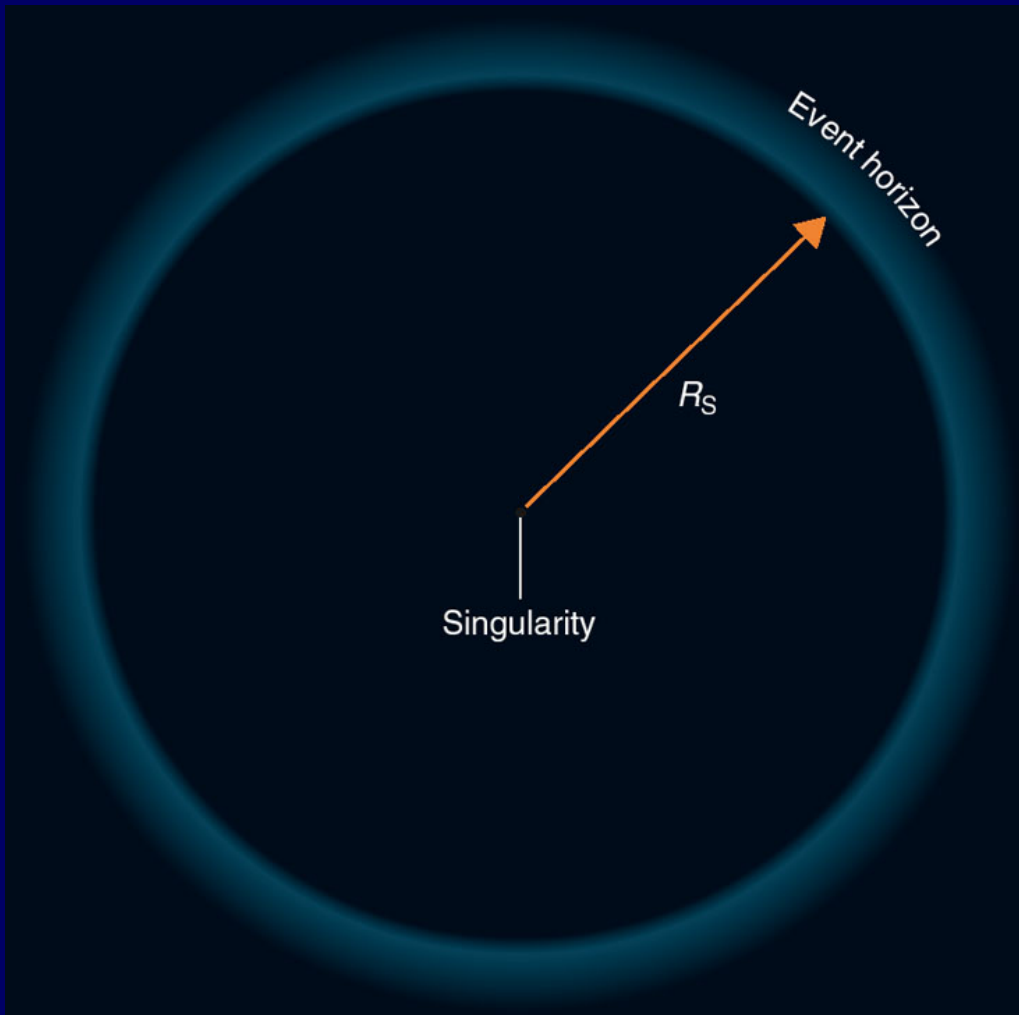
$R_s$  is called the Schwarzschild radius.

Note: this formula is true only when all the mass is inside  $R_s$



$v_{esc} = c$

# Schwarzschild Radius and the Event Horizon



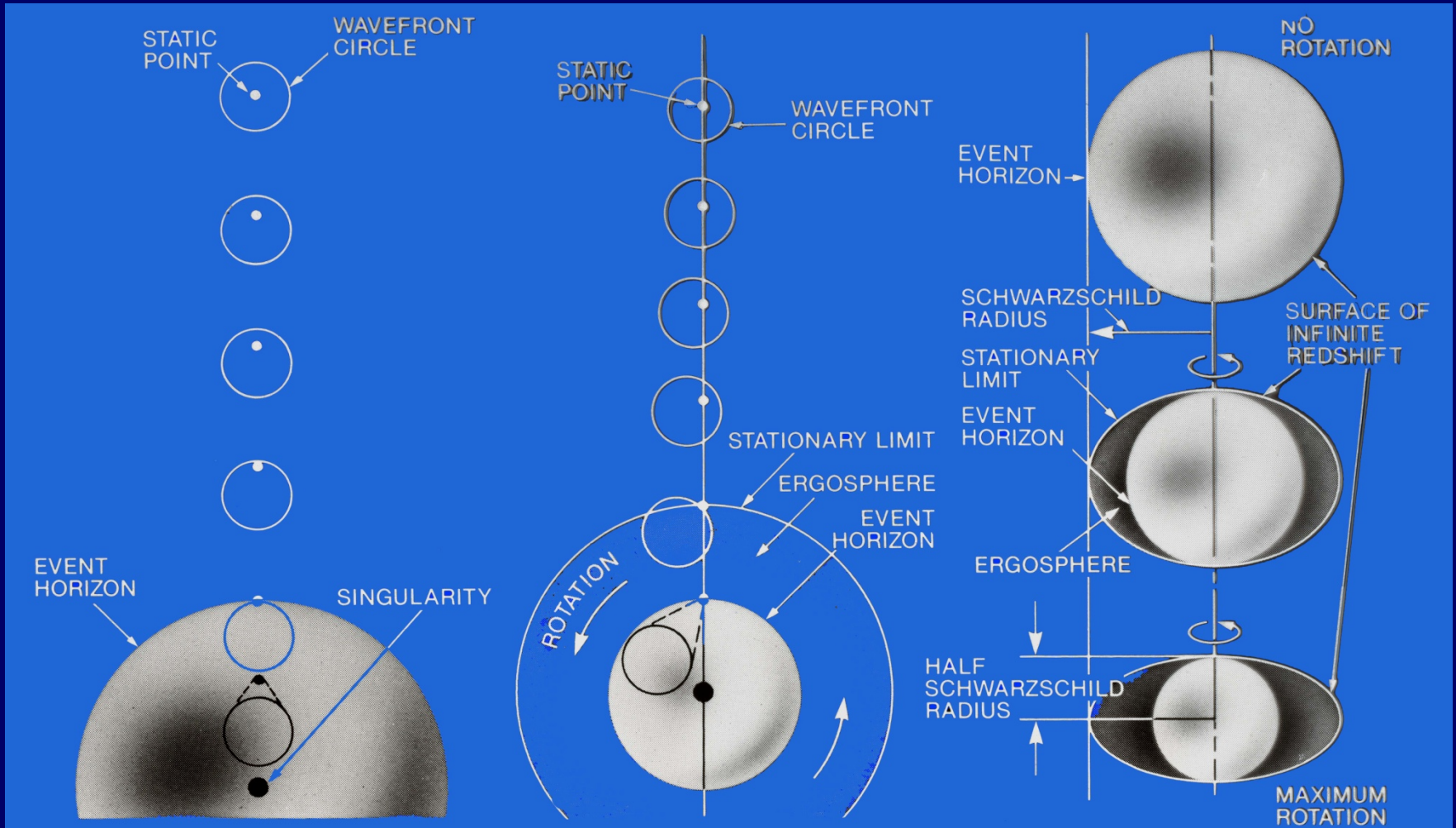
No object can travel faster than the speed of light

⇒ nothing (not even light) can escape from inside the Schwarzschild radius

⇒ We have no way of finding out what's happening inside the Schwarzschild radius.

⇒ **Event horizon**

# Effect of Stationary and Rotating Black Holes on Light Wavefronts



**Schwarzschild (stationary)  
Black Hole**

**Kerr (rotating)  
Black Hole**

# **“Black Holes Have No Hair”**

**The property of matter forming a black hole becomes unknowable because it is all within the event horizon.**

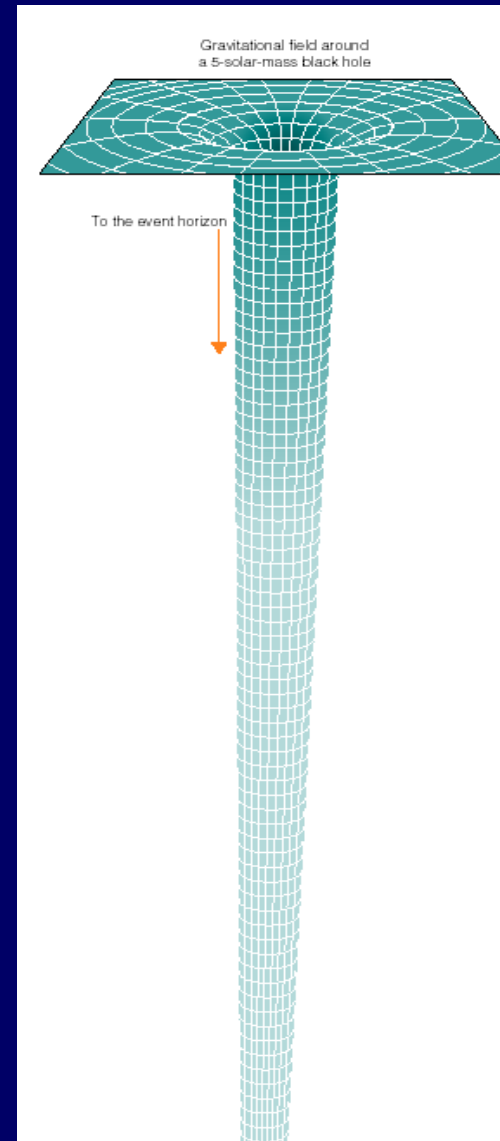
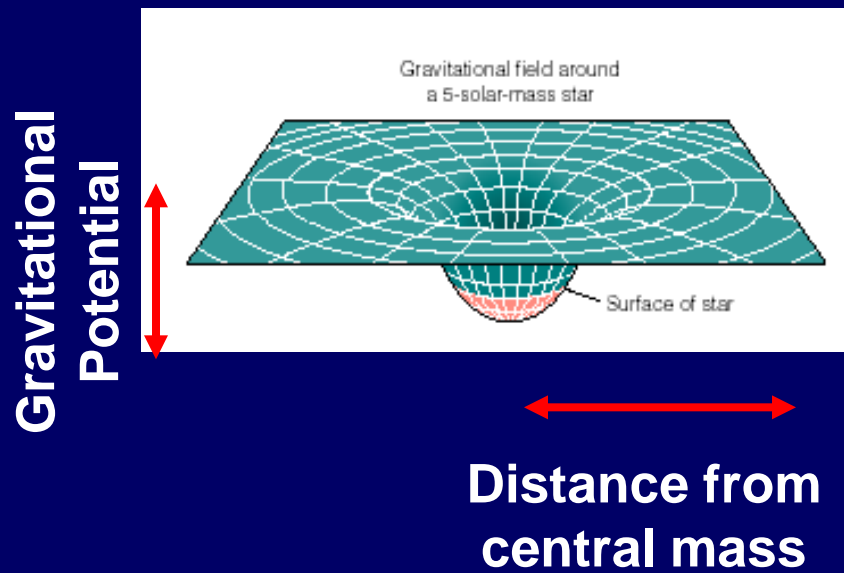
**As a result, black holes have only 3 observable properties that can be measured externally:**

**mass**

**angular momentum**

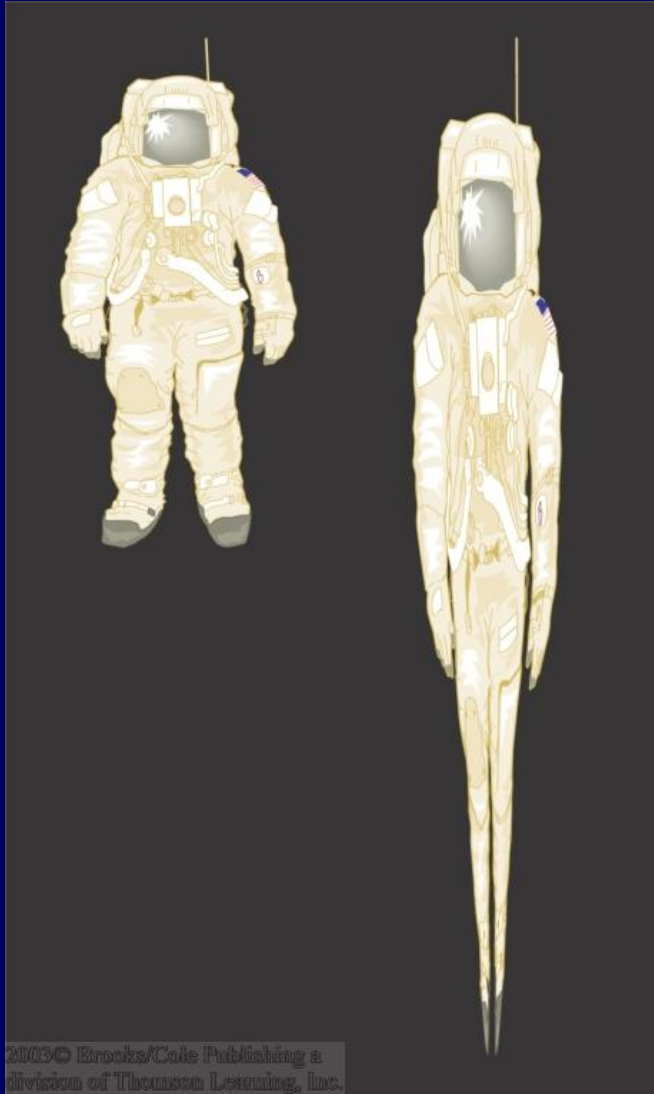
**(electric charge)**

# The Gravitational Field of a Black Hole



The gravitational potential (and gravitational attraction force) at the Schwarzschild radius of a black hole becomes **infinite**.

# General Relativity Effects Near Black Holes



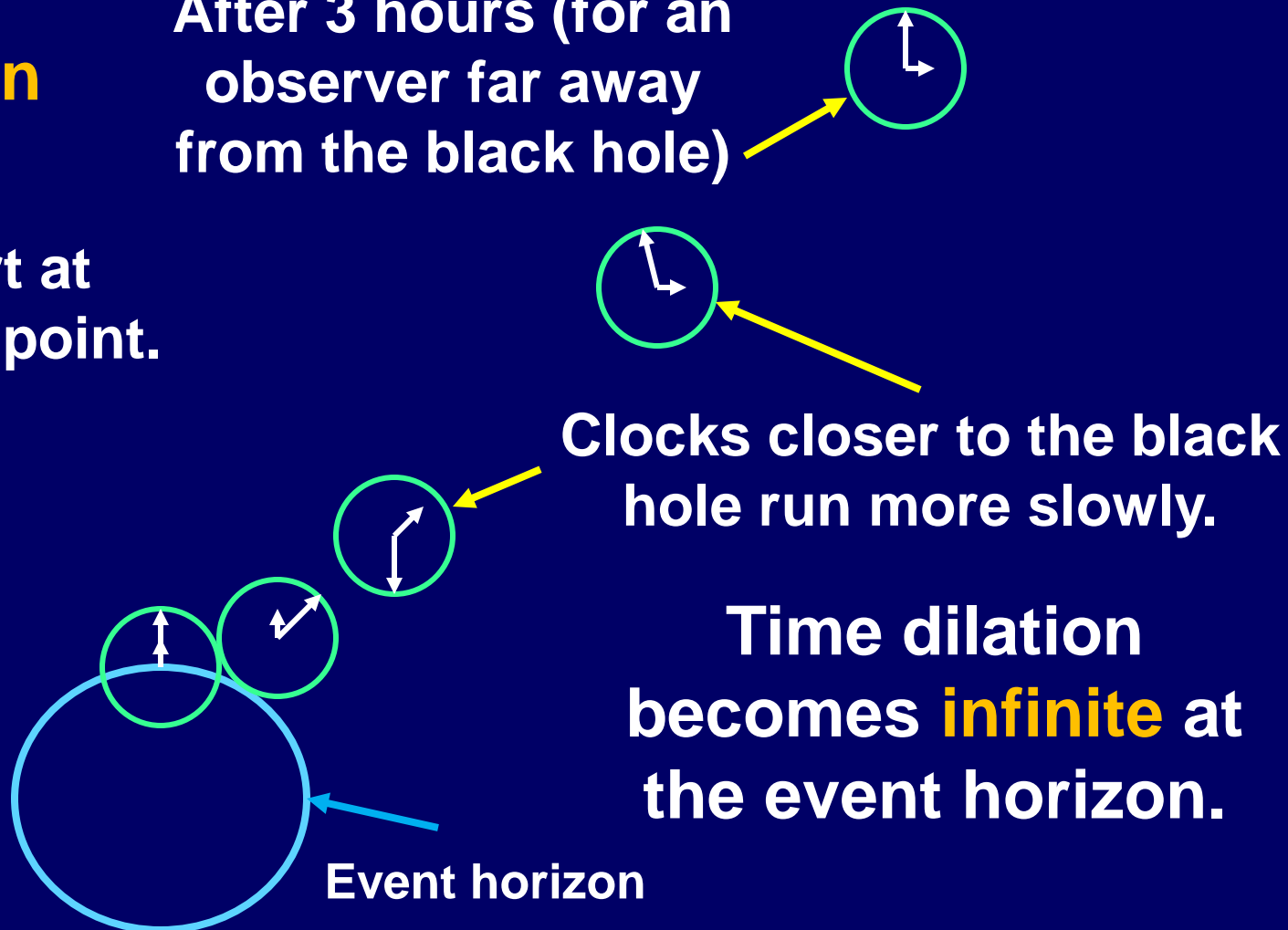
**A body descending toward the event horizon of a black hole will be stretched vertically (tidal effects) and squeezed laterally.**

# General Relativity Effects Near Black Holes

## Time dilation

After 3 hours (for an observer far away from the black hole)

Clocks start at 12:00 at each point.



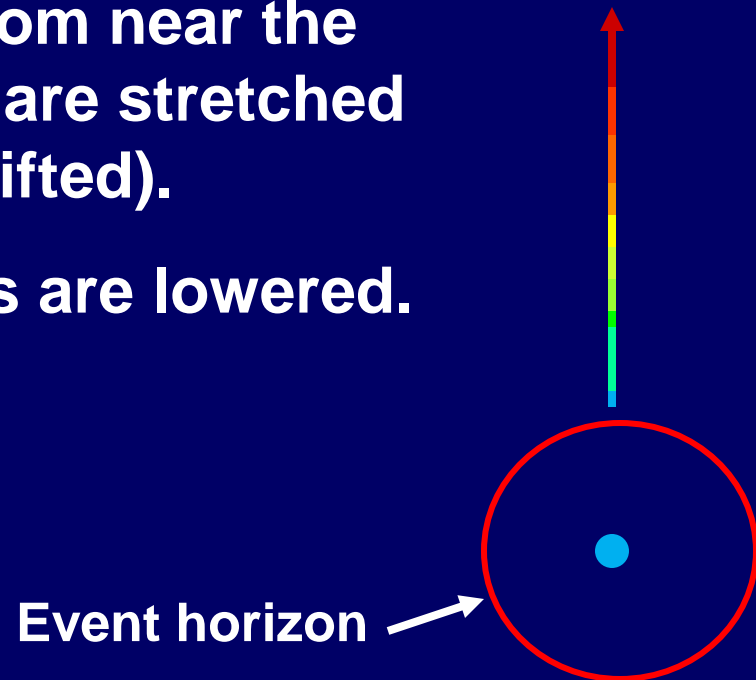


# General Relativity Effects Near Black Holes

## Gravitational Redshift

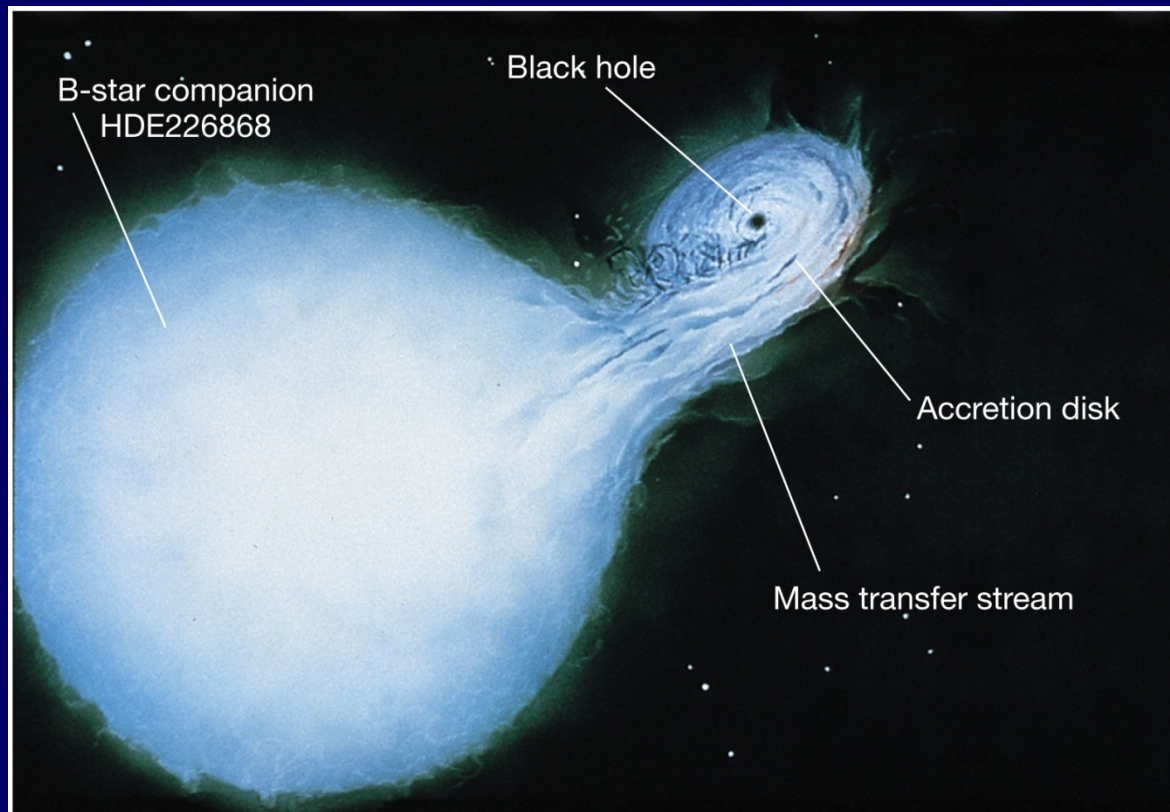
All wavelengths of  
emissions from near the  
event horizon are stretched  
(redshifted).

⇒ Frequencies are lowered.



# Observational Evidence for Stellar Black Holes

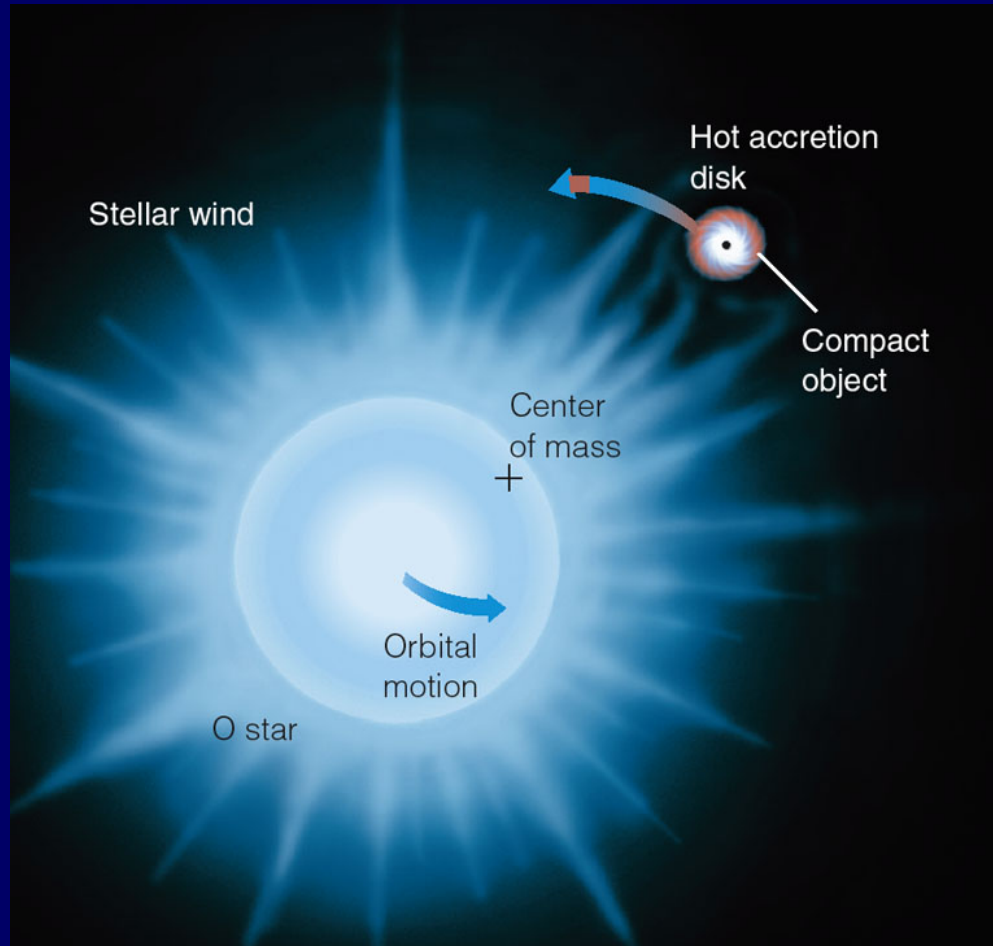
The existence of black-hole binary partners for ordinary stars can be inferred by the effect the hole has on the star's orbit or by radiation from in falling matter.



# Observing Stellar Black Holes

Light cannot escape a black hole

⇒ Black holes can not be observed directly.



If an invisible compact object is part of a binary, we can measure its mass from the orbital period and radial velocity (Kepler's 3<sup>rd</sup> Law).

**Mass >  $\sim 3 M_{\odot}$**


**⇒ Black Hole!**

# Stellar Black Hole Candidates

■ Table 11-2 | Nine Black Hole Candidates

Object	Location	Companion Star	Orbital Period	Mass of Compact Object
Cygnus X-1	Cygnus	O supergiant	5.6 days	$>3.8 M_{\odot}$
LMC X-3	Dorado	B3 main-sequence	1.7 days	$\sim 10 M_{\odot}$
A0620-00	Monocerotis	K main-sequence	7.75 hours	$10 \pm 5 M_{\odot}$
V404 Cygni	Cygnus	K main-sequence	6.47 days	$12 \pm 2 M_{\odot}$
J1655-40	Scorpius	F-G main-sequence	2.61 days	$6.9 \pm 1 M_{\odot}$
QZ Vul	Vulpecula	K main-sequence	8 hours	$10 \pm 4 M_{\odot}$
4U 1543-47	Lupus	A main-sequence	1.123 days	$2.7-7.5 M_{\odot}$
V4641 Sgr	Sagittarius	B supergiant	2.81678 days	$8.7-11.7 M_{\odot}$
XTEJ1118+480	Ursa Major	K main-sequence	0.170113 days	$>6 M_{\odot}$

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**Compact object  
with  $> \sim 3 M_{\odot}$  must  
be a black hole!**