10.36. (a) Calculate the magnitude of the angular momentum of the Earth in a circular orbit around the Sun. Is it reasonable to model it as a particle? (b) Calculate the magnitude of the angular momentum of the Earth due to its rotation around an axis through the north and south poles, modeling it as a uniform sphere.

$m_{\mathrm{E}} = 5.97\times10^{24}\ \mathrm{kg}$

$R_{\mathrm{E}} = 6.38\times10^{6}\ \mathrm{m}$

Orbital radius $r = 1.50\times10^{11}\ \mathrm{m}$

Period of rotation P_{rot} = $24\ \mathrm{h} = 86{,}400\ \mathrm{s}$

Period of revolution P_{rev} = $1\ \mathrm{y} = 3.156\times10^{7}\ \mathrm{s}$

Identify: $L_z = I\omega_z$

Set Up: For a particle of mass *m* moving in a circular path at a distance *r* from the axis, $I = mr^2$ and $v = r\omega$. For a uniform sphere of mass *M* and radius *R* and an axis through its center, $I = \frac{2}{5}MR^2$. The earth has mass $m_{\mathrm{E}} = 5.97\times10^{24}\ \mathrm{kg}$, radius $R_{\mathrm{E}} = 6.38\times10^{6}\ \mathrm{m}$ and orbit radius $r = 1.50\times10^{11}\ \mathrm{m}$. The earth completes one rotation on its axis in $24\ \mathrm{h} = 86{,}400\ \mathrm{s}$ and one orbit in $1\ \mathrm{y} = 3.156\times10^{7}\ \mathrm{s}$.

Execute:

(a) $$L_z = I\omega_z = mr^2\omega_z = (5.97\times10^{24}\ \mathrm{kg})(1.50\times10^{11}\ \mathrm{m})^2\left(\frac{2\pi\ \mathrm{rad}}{3.156\times10^{7}\ \mathrm{s}}\right) = 2.67\times10^{40}\ \mathrm{kg\cdot m^2/s}.$$

The radius of the earth is much less than its orbit radius, so it is very reasonable to model it as a particle for this calculation.

(b) $$L_z = I\omega_z = \left(\tfrac{2}{5}MR^2\right)\omega = \tfrac{2}{5}(5.97\times10^{24}\ \mathrm{kg})(6.38\times10^{6}\ \mathrm{m})^2\left(\frac{2\pi\ \mathrm{rad}}{86{,}400\ \mathrm{s}}\right) = 7.07\times10^{33}\ \mathrm{kg\cdot m^2/s}$$

Evaluate: The angular momentum associated with each of these motions is very large. Note that the orbital angular momentum is over 7 orders of magnitude greater than its rotational angular momentum. In fact, most of the total angular momentum of the solar system comes from the sum of the orbital angular momenta of its planets.