# Cooperative Collision Avoidance between Multiple Mobile Robots

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This paper presents a new collision avoidance technique, called cooperative collision avoidance, for multiple mobile robots. The detection of the danger of collision between two mobile robots is discussed with respect to the geometric aspects of their paths as are cooperative collision avoidance behaviors. The direction control command and the velocity control command for the cooperative collision avoidance are then proposed. The avoidance technique is extended to cases in which the number of mobile robots is more than two. Furthermore, the conditions for collision avoidance are considered with respect to the navigation parameters and guidelines of designing the navigation parameters are obtained. The effectiveness of the proposed technique is demonstrated by means of numerical simulation and navigation experiments using two real mobile robots named Pioneer-1.

## I. INTRODUCTION

Mobile robots have been the subject of much research and development due to their applicability in a wide variety of tasks in industry, human supported works, and elsewhere. If the task requested cannot be easily carried out by a single robot, multiple robots should be cooperatively used.[1–4] Since the use of multiple robots may lead to collision, their navigation and collision avoidance have been discussed from various points of view. In Ref. [1], for example, a parallel processing system was con-

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structed for the cellular robotic system and was applied to the behavior decision of multiple mobile robots. Mataric[5,6] proposed avoidance behavior rules when a collision between two mobile robots was predicted: stopping the robots for a fixed period or changing their directions. Arkin[7] and Sugihara *et al.*[8] proposed algorithms combining these rules. Shan and Hasegawa[9] proposed behavior-based motion planning of multiple mobile robots in a narrow passage. On the other hand, learning techniques were incorporated into neural networks and/or fuzzy logic controllers for mobile robot navigation.[10,11] Iterating navigation of mobile robots in a given environment, navigation parameters associated with robot's behaviors were adjusted by the learning techniques. These approaches did not give analytical discussions for selecting the navigation parameters although the effectiveness was shown in simulation and navigation experiments. Therefore, the initial selection of the navigation parameters remains to be solved.

This paper presents the navigation of multiple mobile robots with collision avoidance in a two-dimensional free-space environment and discusses collision avoidance conditions from the theoretical points of view. Each robot has its own goal or position to be navigated. The objective is to navigate the multiple mobile robots to their goals without collisions between them. To do this, this paper proposes a new collision avoidance technique called *cooperative collision avoidance* and gives the avoidance conditions with respect to the navigation parameters. The navigation scheme used in this paper is based on the *adaptive navigation technique*[12] in which the navigation law is given by a first-order differential equation, and the navigation of the robot to the goal and obstacle avoidance are done by switching the direction angle command adaptively. In this paper, the cooperative collision avoidance behaviors of the multiple mobile robots are then incorporated into the adaptive navigation technique.

The remainder of this paper is organized as follows. The statements of problem are presented in Section II. The cooperative collision avoidance is discussed in Section III. The detection of collision between two mobile robots is first discussed with respect to aspects under which the two robots encounter each other. The direction control command and the velocity control command for the cooperative collision avoidance are then proposed. The avoidance technique is extended to cases in which the number of mobile robots is more than two. The conditions for collision avoidance are considered with respect to the navigation parameters in Section IV. According to the conditions, a design procedure of the navigation parameters is given. The effectiveness of the proposed technique is demonstrated by means of numerical simulation examples in Section V. In Section VI, the proposed technique is applied to an experimental robot named *Pioneer-1*. The conclusion is presented in Section VII.

## II. STATEMENT OF PROBLEM

This section describes a navigation problem of multiple mobile robots in a two-dimensional free-space environment. Let $n_r$ be the number of mobile robots. The $i$th mobile robot is denoted as $R_i$ and its position is represented by the cartesian coordinates $(x_i(t), y_i(t))$ where $t$ is time. The direction angle of the $i$th robot is $\theta_i(t)$, $(-\pi \le \theta_i(t) < \pi\,[\mathrm{rad}])$, which is measured from the $x$-axis. The velocity of the $i$th robot is $v_i(t)$. The equations of motion of the $n_r$ mobile robots are then given by

$$\begin{aligned} \dot{x}_i(t) &= v_i(t)\cos\theta_i(t) \\ \dot{y}_i(t) &= v_i(t)\sin\theta_i(t) \end{aligned} \qquad (i=1,\ldots,n_r). \tag{1}$$

The objective is to navigate the $n_r$ mobile robots from their starting points $S_i$ $(i=1,\ldots,n_r)$ to their own goals $G_i$ $(i=1,\ldots,n_r)$ without any collisions between them. The following assumption is given on the robots.

**(A1)** All of mobile robots have the same size and the same performance. The radius of the robots is $r_R$.

**(A2)** $(x_i(t), y_i(t), \theta_i(t), v_i(t))$ $(i=1,\ldots,n_r)$ are known to each other by some means of communication.

The direction angle of the mobile robots is given by

$$\dot{\theta}_i(t) = -\eta_\theta[\theta_i(t) - \theta_i^*(t)] \qquad (i=1,\ldots,n_r), \tag{2}$$

where $\eta_\theta$ is a positive constant. This process can be used not only to navigate mobile robots to their goal, but also to avoid collisions by switching the direction angle command $\theta_i^*(t)$.[12] Equation (2) is called the *direction control*.

Similarly, the velocity of the mobile robots is given by

$$\dot{v}_i(t) = -\eta_v[v_i(t) - v_i^*(t)] \qquad (i = 1, \ldots, n_r), \quad (3)$$

where

$$0 \leq v_{\min} \leq v_i^*(t) \leq v_{\max}. \quad (4)$$

$\eta_v$ is a positive constant, $v_i^*(t)$ is the velocity command, $v_{\min}$ is the minimum velocity, and $v_{\max}$ is the maximum velocity. Equation (3) is called the *velocity control.*

Based on the adaptive navigation technique,[12] the direction angle command $\theta_i^*(t)$ and the velocity command $v_i^*(t)$ are given according to the following three modes:

- **Navigation mode:** Navigate a robot to its goal if no collision is detected and the robot is not at goal. Letting $\phi_i(t)$ be the angle from the position of the $i$th robot $R_i$ to its goal $G_i$, $\theta_i^*(t)$ is given by $\theta_i^*(t) = \phi_i(t)$, while $v_i^*(t)$ is given by $v_i^*(t) = v_0$, where $v_0$ is the navigation velocity.
- **Cooperative avoidance mode:** Make a robot avoid collisions with other robots if collisions are predicted. $\theta_i^*(t)$ and $v_i^*(t)$ will be given in the later sections.
- **Final mode:** Approach a robot to its goal when the robot is *near* the goal. Let $l_{G_i}$ be the distance between the $i$th robot $R_i$ and its goal $G_i$. *Near* means that $l_{G_i} < d_f$, where $d_f$ is a range to the final mode. When navigating robot $R_i$ to goal $G_i$, let $t_f$ be the time in which $l_{G_i}$ becomes $l_{G_i} = d_f$ first. Then, $\theta_i^*(t)$ is fixed as $\theta_i^*(t) = \theta_i(t_f)$ for $t \geq t_f$. $v_i^*(t)$ is decreased to zero while approaching to the goal.

The details of the navigation mode and the final mode are described in Ref. [12]. The rest of this paper therefore deals with the cooperative avoidance mode.

## III. COOPERATIVE COLLISION AVOIDANCE

This section presents the collision avoidance between multiple mobile robots, that is, the cooperative avoidance mode. First, a concept of the cooperative collision avoidance is described with two mobile robots, that is, the collision detection, the cooperative collision avoidance behaviors, and the control commands for the avoidance. The cooperative collision avoidance between two mobile robots is then extended to cases in which the number of mobile robots is more than two.

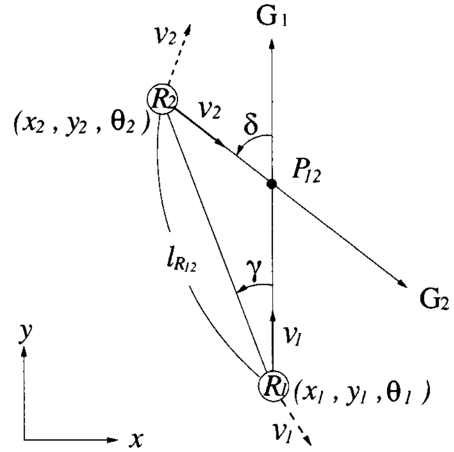

**Figure 1.** Two mobile robots.

### A. Collision Detection and Mode Selection

Figure 1 shows a situation where the position, direction angle, and velocity of two mobile robots $R_1$ and $R_2$ are $(x_i(t), y_i(t))$, $\theta_i(t)$, and $v_i(t)$ $(i = 1, 2)$, respectively. If the robots face in the direction of the dashed arrows, a collision will not happen. However, if the robots face in the direction of the solid arrows, the two mobile robots may collide with each other at a point $P_{12}$, which is called the predicted crossing point. Let $l_{R_{ij}}$ be the distance between robots $R_i$ and $R_j$. In Fig. 1, $l_{R_{12}}$ and $dl_{R_{12}}/dt$ are given by

$$l_{R_{12}} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (5)$$

$$\frac{dl_{R_{12}}}{dt} = \frac{1}{l_{R_{12}}}[(v_2 \cos\theta_2 - v_1 \cos\theta_1)(x_2 - x_1) + (v_2 \sin\theta_2 - v_1 \sin\theta_1)(y_2 - y_1)]. \quad (6)$$

Using them, the danger of collision is classified as Table 1. Evaluating the danger of collision, it is seen that an avoidance behavior is required for Case 4. Thus, $l_{R_{12}}$ and $dl_{R_{12}}/dt$ can be used as a switch to the cooperative avoidance mode. Using $l_{G_i}$, $l_{R_{ij}}$, and $dl_{R_{ij}}/dt$, the mode selection of robot $R_i$ is given as

**Table 1.** Danger of collision with respect to $l_R$ and $dl_R/dt$.

| Case | $l_R$ | $dl_R/dt$ | Danger of collision |
|---|---|---|---|
| 1 | long | $>0$ | Danger is small and is going to be decreased. |
| 2 | long | $<0$ | Danger is small but is going to be increased. |
| 3 | short | $>0$ | Danger is great but is going to be decreased. |
| 4 | short | $<0$ | Danger is great and is going to be increased. |

follows:

```
if $l_{G_i} < d_f$
    then final mode is selected.
else if $l_{R_{ij}} < d_p$ and $dl_{R_{ij}}/dt < -d_v$
    then cooperative avoidance mode is selected.
    else navigation mode is selected.
end if
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$d_p(> 0)$ and $d_v(> 0)$ are, respectively, the distance- and the derivative-switch to the cooperative avoidance mode to be designed.

### B. Cooperative Collision Avoidance Behaviors

As described in Section II, mobile robots have two control strategies, the direction angle control and the velocity control for navigation and collision avoidance. The purpose of this section is to present an avoidance technique using these strategies for the cooperative collision avoidance.

Let $\delta$ and $\gamma$ $(-\pi \leq \delta, \gamma < \pi)$ be defined as the crossing angle and the relative angle respectively as shown in Fig. 1. Using the crossing angle $\delta$, aspects under which two mobile robots may encounter each other in a two-dimensional free-space environment are classified as given in Fig. 2. In Fig. 2(a), a collision will occur unless the robots change their directions. In this case, the direction control is required to avoid collision. The velocity control, however, is of no use. In Fig. 2(c), on the other hand, a collision may be avoided by changing the directions of the two robots, or changing their velocities so that they do not go through the predicted crossing point at the same time. From the viewpoint of safety and control, the velocity control is more effective in avoiding collision than the direction control. When using the velocity control, a decision has to be made as to which robot should go through the predicted crossing point first. In this paper, two *priorities* of two mobile robots, *high priority* and *low priority*, are first defined, and the behaviors of the two mobile robots are then decided according to these priorities. As an example, in Fig. 2(c), the robot with the high priority is accelerated to go through the predicted crossing point first, while the other robot with the low priority is decelerated.

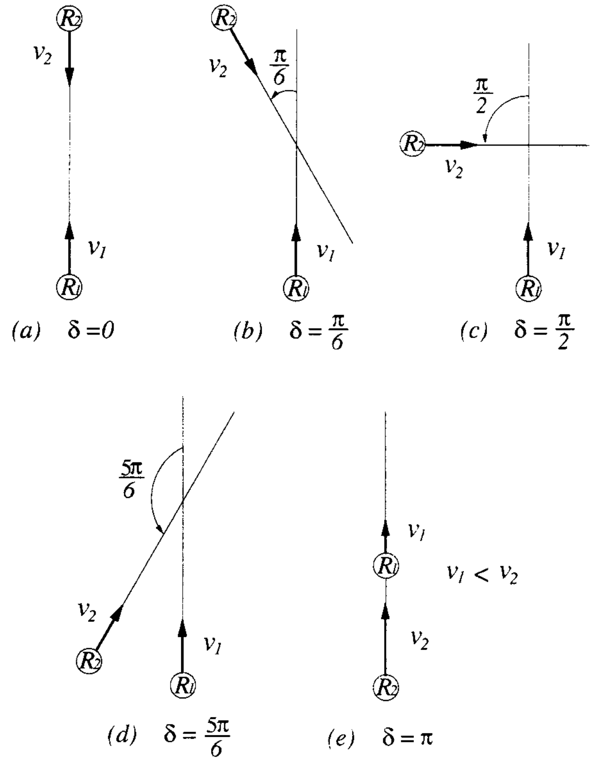


**Figure 2.** Aspects under which two mobile robots encounter each other.

In Fig. 2(e), where $v_2 > v_1$, both the direction control and the velocity control should be used to avoid a collision. For example, the direction of robot $R_2$ could be changed and its velocity increased, while the direction of robot $R_1$ could be kept constant and its velocity decreased. These responses tend to reduce the danger of collision as quickly as possible.

Figure 2(b) is an intermediate aspect between Figs. 2(a) and 2(c), while Fig. 2(d) is between Figs. 2(c) and 2(e). For these aspects, the direction control and the velocity control are used interpolatively. The details will be given later.

### C. Definition of Priorities of Two Mobile Robots

As described in the previous section, a predicted collision is avoided so that the robot with the high priority acts prior to the other robot with the low priority. The high priority should be then assigned to a robot that will arrive at the predicted crossing point first. That is, the priorities of two mobile robots are defined as the inverse of their arrival time at the predicted crossing point in this paper. In Figs. 2(b), 2(c), and 2(d), the predicted crossing point $P_{12}$ is calculated by using $(x_i, y_i, \theta_i)$ $(i = 1, 2)$. Denoting the distance between the position of robot $R_i$ and the predicted crossing point as $\overline{R_i P_{12}}$, the priorities $w_i$ $(i = 1, 2)$ are given by

$$w_i = \frac{v_i}{\overline{R_i P_{12}}} \qquad (i = 1, 2). \tag{7}$$

If there is no predicted crossing point because the robots are moving along a straight line such as shown in Figs. 2(a) and 2(e), the priorities are given by

$$w_i = v_i \qquad (i = 1, 2). \tag{8}$$

That is, the robot whose velocity is greater is assigned to the high priority. If the priorities are obtained as $w_1 = w_2$, robot $R_1$ is assigned to the high priority while robot $R_2$ is the low priority in this paper.

### D. Direction Angle Command and Velocity Command

This section presents a design of the direction angle command and the velocity command to realize the cooperative collision avoidance behaviors described in Section III B. Let $\theta_{ci}^*(t)$ be the direction angle command for the cooperative avoidance of robot $R_i$. It is given by

$$\theta_{ci}^* = \theta_i + \Delta\theta_{ci}^*, \tag{9}$$

where $\Delta\theta_{ci}^*$ is an additional angle for the cooperative avoidance mode. As mentioned in Section III B, the cooperative collision avoidance behaviors depend on the priority and the crossing angle $\delta$. According to the priority, $\Delta\theta_{ci}^*$ is defined as

$$\Delta\theta_{ci}^* = \begin{cases} \Delta\theta_{ch}^* & w_i \geq w_j \\ \Delta\theta_{cl}^* & w_i < w_j \end{cases} \qquad i, j = 1 \text{ or } 2, \quad i \neq j, \tag{10}$$

where $\Delta\theta_{ch}^*$ and $\Delta\theta_{cl}^*$ are additional angles of high and low priority. They are given by

$$\Delta\theta_{ch}^* = k_\theta \operatorname{sgn}(\delta)\left|1 - \frac{2}{\pi}|\delta|\right| \tag{11}$$

$$\Delta\theta_{cl}^* = \operatorname{sgn}(\delta)\operatorname{sat}\left(|\delta|, 0, \frac{\pi}{2}, k_\theta, 0\right), \tag{12}$$

where

$$\operatorname{sgn}(x) \triangleq \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases} \tag{13}$$

$$\operatorname{sat}(x, a, b, c, d) \triangleq \begin{cases} c & x < a \\ \dfrac{d - c}{b - a}(x - a) + c & a \leq x < b \\ d & x \geq b. \end{cases} \tag{14}$$

$k_\theta$ is the maximum of $|\Delta\theta_{ci}^*|$, sat( ) is the saturation function and is shown in Fig. 3, and $\Delta\theta_{ch}^*$ and $\Delta\theta_{cl}^*$ are as shown in Fig. 4.

Similarly, let $v_{ci}^*$ be the velocity command for the cooperative avoidance mode. It is given by

$$v_{ci}^* = \begin{cases} v_{ch}^* & w_i \geq w_j \\ v_{cl}^* & w_i < w_j \end{cases} \qquad i, j = 1 \text{ or } 2, \quad i \leq j, \tag{15}$$

where

$$v_{ch}^* = \operatorname{sat}\left(|\delta|, 0, \frac{\pi}{2}, v_{0i}, v_{\max}\right) \tag{16}$$

$$v_{cl}^* = \operatorname{sat}\left(|\delta|, 0, \frac{\pi}{2}, v_{0i}, v_{\min}\right), \tag{17}$$

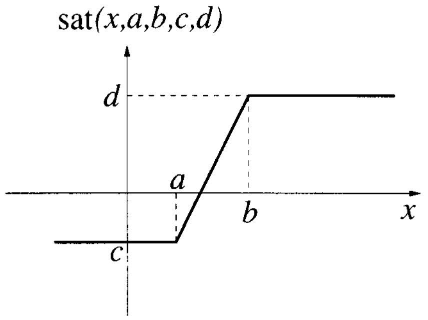


**Figure 3.** Function sat( $x, a, b, c, d$).

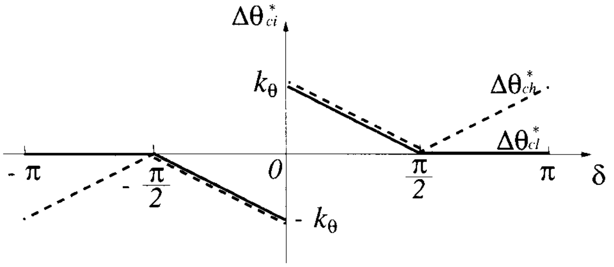


**Figure 4.** Additional angle for cooperative avoidance $\Delta\theta_{ci}^{*}(t)$.

and $v_{ch}^{*}$ and $v_{cl}^{*}$ are, respectively, the velocity commands of high and low priority and are shown in Fig. 5. At $\delta = 0$ [Fig. 2(a)], the direction control is fully used while the velocity control is not used. At $\delta = \pi/2$ [rad] [Fig. 2(c)], on the other hand, only the velocity control is used according to the priority. During $0 \leq \delta < \pi/2$, $\Delta\theta_{ci}^{*}$ and $v_{ci}^{*}$ are linearly interpolated as shown in Figs. 4 and 5. This corresponds to Fig. 2(b).

### E. Extension of Cooperative Collision Avoidance

This section presents an extension of this cooperative collision avoidance to situations where the number of mobile robots is more than two. In situations such as this, it is to be noted that there exist multiple relations between any two mobile robots. A mobile robot has $n_r - 1$ relations with the rest of robots. Therefore, the direction angle command $\theta_{ci}^{*}$ and the velocity command $v_{ci}^{*}$ are decided so as to integrate the multiple relations.

Consider first the situation shown in Fig. 6 where three mobile robots are moving toward each other. By evaluating $l_{R_{ij}}$ and $dl_{R_{ij}}/dt$ $(i, j = 1,2,3,$ $i \neq j)$, the danger of collision is detected at two points, $P_{12}$ and $P_{23}$, where $P_{ij}(=P_{ji})$ is the predicted crossing point of robots $R_i$ to $R_j$. Let $w_{ij}$ be the priority of robot $R_i$ relative to robot $R_j$. When the danger of collision of robot $R_i$ with robot $R_j$ is detected, the priority $w_{ij}$ is given by

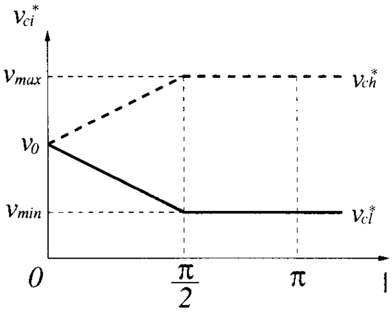


**Figure 5.** Velocity command for cooperative avoidance $v_{ci}^{*}$.

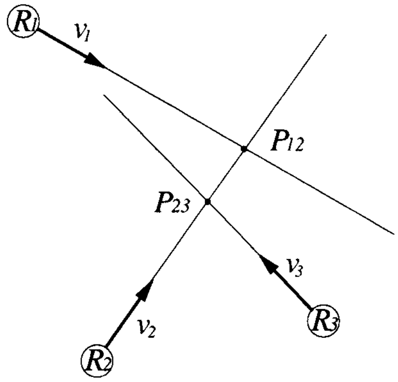


**Figure 6.** Three mobile robots.

$$w_{ij} = \frac{v_i}{\overline{R_i P_{ij}}}, \tag{18}$$

where $w_{ii}$ is set to $w_{ii} = 0$. If $P_{ij}$ is not found, $w_{ij} = v_i$. Moreover, if the danger of collision is not detected, $w_{ij} = 0$. Equation (18) is an extended expression of Eq. (7). Table 2 shows the priorities for Fig. 6. The avoidance behaviors of robots $R_1$ and $R_3$ are respectively decided according to the single relation with respect to robot $R_2$. On the other hand, robot $R_2$ has two relations with respect to robots $R_1$ and $R_3$. It is preferable that the behavior of robot $R_2$ is given by evaluating all the relations in which it is involved. Then, the avoidance behavior of robot $R_i$ is determined by taking the center of gravity whose weight is the priority $w_{ij}$. Then, $\theta_{ci}^{*}$

**Table 2.** Weights between three mobile robots of Fig. 6.

| | Opponent | | |
|---|---|---|---|
| | $R_1$ | $R_2$ | $R_3$ |
| $R_1$ | 0 | $w_{12}$ | 0 |
| $R_2$ | $w_{21}$ | 0 | $w_{23}$ |
| $R_3$ | 0 | $w_{32}$ | 0 |

and $v_{ci}^*$ are given by

$$\theta_{ci}^* = \frac{\sum_{j=1}^{n_r} w_{ij}\theta_{cij}^*}{\sum_{j=1}^{n_r} w_{ij}} \tag{19}$$

$$v_{ci}^* = \frac{\sum_{j=1}^{n_r} w_{ij}v_{cij}^*}{\sum_{j=1}^{n_r} w_{ij}}, \tag{20}$$

where $\theta_{cij}^*$ and $v_{cij}^*$ are the direction angle command and the velocity command for the cooperative collision avoidance of robot $R_i$ with robot $R_j$, respectively.

## IV. COLLISION AVOIDANCE CONDITION AND NAVIGATION PARAMETERS

As mentioned so far, there are several navigation parameters associated with mobile robots, avoidance behaviors, and mode selection. They are classified as follows:

- **Robot parameter:** $r_R, v_0, \eta_\theta, \eta_v$
- **Avoidance parameter:** $k_\theta, v_{\max}, v_{\min}$
- **Mode selection parameter:** $d_p, d_v$

The robot parameters are associated with mobile robots themselves and are given in advance. The avoidance parameters are associated with the cooperative collision avoidance behaviors and remain to be designed. The mode selection parameters are also to be designed. This section first discusses the conditions for collision avoidance between two mobile robots with the navigation parameters. The conditions are used for designing the avoidance and the mode selection parameters. For convenience in the following sections, $d_p$ is replaced by a non-dimensional parameter $l_p$ defined as

$$l_p \triangleq \frac{d_p}{2r_R}. \tag{21}$$

### A. Collision Avoidance Condition

Although collision avoidance conditions derived in this section are for typical avoidance cases whose predicted crossing angle $\delta$ is 0 and $\pi/2$ [rad], they are used as guidelines for designing the navigation parameters. The following assumptions are given to derive the avoidance conditions:

**(A3)** Let $t_0$ be the time when the cooperative avoidance mode is selected. The velocities of two mobile robots are

$$v_1(t_0) = v_2(t_0) = v_0. \tag{22}$$

**(A4)** The navigation velocity $v_0$ is given by

$$v_0 = \frac{v_{\max} + v_{\min}}{2}. \tag{23}$$

**(1)** $\delta = 0$. Under assumptions (A1)–(A3), a condition for collision avoidance of $\delta = 0$ [rad] is given as follows.

**Theorem 1:** *Consider an aspect under which two mobile robots encounter each other with the predicted crossing angle $\delta = 0$ [rad]. Then, a collision between the robots is avoided only if the following relation is held during the cooperative avoidance mode:*

$$\frac{1}{1 + A_\theta} > \cos\{(l_p - 1)A_\theta\} \tag{24}$$

*where*

$$A_\theta \triangleq \frac{\eta_\theta k_\theta r_R}{v_0}. \tag{25}$$

*Proof:* At $\delta = 0$, only the direction control is used. Figure 7 shows a situation in which two mobile robots approach each other closest. The trajectories of the robots are symmetric with respect to the predicted crossing point $P_{12}$. During the cooperative avoidance mode, $\delta$ is kept at $\delta = 0$ [rad]. From

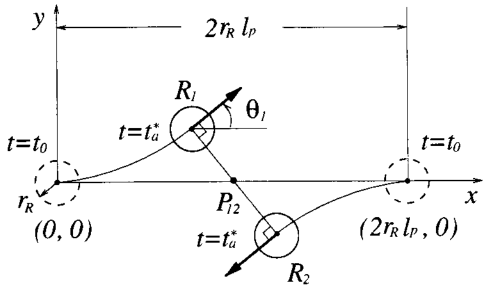


**Figure 7.** Cooperative avoidance for $\delta = 0$ [rad].

Eqs. (9)–(12), the direction angle command for the cooperative collision avoidance $\theta_{ci}^{*}$ is given by

$$\theta_{ci}^{*} = \theta_i + k_\theta \qquad (i = 1, 2). \tag{26}$$

Substituting Eq. (26) into Eqs. (1) and (2) and integrating them, we have

$$\theta_i(t) = \eta_\theta k_\theta (t - t_0) + \theta_i(t_0) \tag{27}$$

$$\begin{aligned} x_i(t) &= \frac{v_0}{\eta_\theta k_\theta}\{\sin\theta_i(t) - \sin\theta_i(t_0)\} + x_i(t_0) \\ y_i(t) &= -\frac{v_0}{\eta_\theta k_\theta}\{\cos\theta_i(t) - \cos\theta_i(t_0)\} + y_i(t_0) \end{aligned} \tag{28}$$

$(i = 1, 2)$.

Without loss of generality, $t_0$ is given by $t_0 = 0$. The starting points of robots $R_1$ and $R_2$ can be coordinated as

$$(x_1(t_0), y_1(t_0), \theta_1(t_0)) = (0, 0, 0), \tag{29}$$

$$(x_2(t_0), y_2(t_0), \theta_2(t_0)) = (2r_R l_p, 0, -\pi). \tag{30}$$

If $r_R < \overline{R_1 P_{12}} = \overline{R_2 P_{12}}$ is satisfied during the cooperative avoidance mode, a collision is avoided. Let $t_a^{*}$ be the time when the distance of robots $R_1$ and $R_2$ is the shortest during the cooperative avoidance mode and let $t_a$ be the time when the two mobile robots collide with each other without changing their directions; that is,

$$t_a = \frac{x_1(t_a)}{v_0} = \frac{(l_p - 1) r_R}{v_0}. \tag{31}$$

During the cooperative avoidance mode, the following relations are held:

$$t_a \le t_a^{*}, \qquad y_1(t_a) \le y_1(t_a^{*}), \qquad \cos\theta_1(t_a) \ge \cos\theta_1(t_a^{*}) \tag{32}$$

Using Eqs. (27) and (28), a sufficient condition for $r_R < \overline{R_1 P_{12}} = y_1(t_a^{*})/\cos\theta_1(t_a^{*})$ is given by

$$r_R < \frac{y_1(t_a)}{\cos\theta_1(t_a)} = \frac{v_0}{\eta_\theta k_\theta}\left\{\frac{1}{\cos\theta_1(t_a)} - 1\right\}. \tag{33}$$

Defining $A_\theta$ as Eq. (25), Eq. (33) is written as Eq. (24). ■

In Eq. (25), $\eta_\theta$, $k_\theta$, and $r_R$ are proportional to $A_\theta$, while $v_0$ is inversely proportional. If the navigation velocity $v_0$ is increased, the rotational acceleration of the direction control, that is, $\eta_\theta$ and $k_\theta$, must be increased to keep a specified $A_\theta$.

(2) $\delta = \pi/2$. Under assumptions (A1)–(A4), a condition for collision avoidance of $\delta = \pi/2$ [rad] is given as follows.

**Theorem 2:** *Consider an aspect under which two mobile robots encounter each other with the predicted crossing angle $\delta = \pi/2$ [rad]. Then, a collision between the robots is avoided only if the following relation is held during the cooperative avoidance mode,*

$$g(t_b, \eta_v) > \frac{r_R}{\Delta v}, \tag{34}$$

*where*

$$\Delta v \triangleq \frac{v_{\max} - v_{\min}}{2} \tag{35}$$

$$g(t_b, \eta_v) \triangleq t_b - \frac{1}{\eta_v}(1 - e^{-\eta_v t_b}). \tag{36}$$

*$t_b$ is the time which satisfies the following equation:*

$$\sqrt{2}\, r_R l_p - v_{\max} t_b + \frac{\Delta v}{\eta_v}(1 - e^{-\eta_v t_b}) = 0. \tag{37}$$

*Proof:* At $\delta = \pi/2$, only the velocity control is used. Figure 8 shows a situation in which two mobile robots $R_1$ and $R_2$ are encountered where $\delta$ is $\pi/2$ [rad]. At $t = t_0$, the starting points of robots $R_1$ and $R_2$ can be respectively coordinated as $(x_{10}, 0, -\pi)$

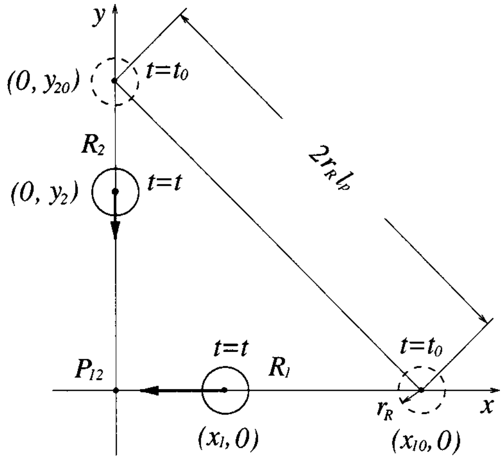


**Figure 8.** Cooperative avoidance for $\delta = \pi/2$ [rad].

and $(0, y_{20}, -\pi/2)$, where $x_{10}$, $y_{20} > 0$. The predicted crossing point $P_{12}$ is then at the origin. If $x_{10} \le y_{20}$ and $v_1(t_0) = v_2(t_0) = v_0$, the relation of the priority of the two robots is given by $w_1 \ge w_2$. For $t > t_0(=0)$, the velocities of the two robots are

$$v_1(t) = v_{\max} - \Delta v e^{-\eta_v t} \tag{38}$$

$$v_2(t) = v_{\min} + \Delta v e^{-\eta_v t}. \tag{39}$$

The positions of the robots are given by

$$\begin{aligned} x_1(t) &= x_{10} - \int_0^t v_1(\tau)\, d\tau \\ &= x_{10} - v_{\max} t + \frac{\Delta v}{\eta_v}(1 - e^{-\eta_v t}) \end{aligned} \tag{40}$$

$$\begin{aligned} y_2(t) &= y_{20} - \int_0^t v_2(\tau)\, d\tau \\ &= y_{20} - v_{\min} t - \frac{\Delta v}{\eta_v}(1 - e^{-\eta_v t}). \end{aligned} \tag{41}$$

The worst case for collision avoidance is that $y_{20} - x_{10} = 0_+$ and $x_{10} = \sqrt{2}\, r_R l_p$. For this case, letting $t_b$ be the time when robot $R_1$ arrives at the predicted crossing point $P_{12}$, $t_b$ satisfies Eq. (37). If $2r_R < y_2(t_b) - x_1(t_b)$ is satisfied during the cooperative avoidance mode, the collision is avoided. Using $g(t_b, \eta_v)$ defined by Eq. (36), we have

$$2r_R < y_2(t_b) - x_1(t_b) = 2\Delta v g(t_b, \eta_v) + y_{20} - x_{10}. \tag{42}$$

A sufficient condition for Eq. (42) is given by Eq. (34). ■

For a fixed $r_R$, $g(t_b, \eta_v)$ should be large enough to satisfy Eq. (34). $g(t_b, \eta_v)$ is a monotonously increasing function because

$$\frac{\partial g}{\partial t_b} > 0, \qquad \frac{\partial g}{\partial \eta_v} > 0 \qquad \text{for } t_b, \eta_v > 0. \tag{43}$$

Moreover, from Eq. (37), $l_p$ is also monotonously increasing with respect to $t_b$; we have

$$\frac{\partial l_p}{\partial t_b} > 0, \qquad \text{for } t_b > 0. \tag{44}$$

Therefore, a large $\eta_v$ and a large $l_p$ are preferred to hold Eq. (34). That is, the acceleration of the velocity should be great and the distance-switch to the cooperative avoidance mode $d_p$ $(= 2r_R l_p)$ should be long.

## B. Design of Navigation Parameters

This section presents a design of the navigation parameters $k_\theta$, $v_{\max}$, $v_{\min}$ $d_p$, and $d_v$ by taking into consideration the collision avoidance conditions: Theorem 1 and 2. An objective of mobile robots in this paper is to arrive at their own goals. Although collisions must be avoided, the trajectories of mobile robots should be made short from their starting points to goals as much as possible. Equations (24) and (34) in Theorems 1 and 2 are sufficient conditions for collision avoidance. To prevent from overacting the avoidance behaviors, the navigation parameters are therefore designed by taking the values near the lower bound which satisfy Eqs. (24) and (34) in Theorems 1 and 2. A procedure of designing the navigation parameters is given as follows.

**Step 1.** Give the values of the robot parameters $r_R$, $\eta_\theta$, and $\eta_v$ from the performance of mobile robots used. Decide $v_{\max}$, $v_{\min}$, and $d_v$ from the navigation objectives. $d_f$ is designed so as to satisfy Theorem 1 in Ref. [12].

**Step 2.** Draw function $g(t_b, \eta_v)$ and choose the value of $t_b$ near the lower bound.

**Step 3.** Calculate $l_p$ by using Eq. (37). Obtain $d_p$ using Eq. (21).

**Step 4.** Using $l_p$ obtained in Step 3, draw function $f(A_\theta)$ defined as

$$f(A_\theta) \triangleq \frac{1}{1 + A_\theta} - \cos\{(l_p - 1) A_\theta\}. \tag{45}$$

A positive $f(A_\theta)$ means that Eq. (24) is held. Choose the value of $A_\theta$ near the lower bound.

**Step 5.** Calculate $k_\theta$ by using Eq. (25).

In Theorems 1 and 2, the derivative-switch $d_v$ was not discussed. Since the mode should be switched to the cooperative avoidance mode for Case 4 in Table 1, a candidate of $d_v$ is zero. It should be adjusted by evaluating avoidance performance.

## V. NUMERICAL SIMULATION

This section presents numerical simulation examples to demonstrate the effectiveness of the proposed cooperative avoidance behaviors and collision avoidance conditions for multiple mobile robots. First, two mobile robots cases are shown according to the design procedure of the navigation parameters. Using the navigation parameters designed, general cases in which the number of robots $n_r$ is 3 and 5 are shown next. Assumptions (A1)–(A4) were supposed to be held in these simulation examples.

## A. Two Mobile Robots

**Step 1.** The robot parameters used in the numerical simulation were given as

$$r_R = 0.3, \qquad v_0 = 4.0, \qquad \eta_\theta = 8.488, \qquad \eta_v = 4.244. \tag{46}$$

Moreover, $v_{\max}$, $v_{\min}$, and $d_v$ were given as

$$v_{\max} = 8.0, \qquad v_{\min} = 0.0, \qquad d_v = 0.2. \tag{47}$$

From Ref. [12], $d_f$ was given by 0.471.

**Step 2.** Figure 9 shows the plot of function $g(t_b, \eta_v)$ with $\eta_v = 4.244$. The lower bound of collision avoidance condition Eq. (34) was $r_R/\Delta v = 0.077$. The lower bound was then given as $t_b = 0.22$ in Fig. 9.

**Step 3.** Substituting the obtained $t_b$ into Eq. (37), the lower bound of $l_p$ was calculated as 2.8. To hold Eq. (34), the value of $l_p$ was chosen as $l_p = 3.1$ and $d_p$ was then calculated as 1.86.

Figure 10 shows the trajectories of two mobile robots for $\delta = \pi/2$ [rad] and $l_p = 3.1$ to hold Eq. (34). The trajectories were displayed for a specified period $k$, and the number written in the figures indicates the value of $k$. Figure 11 shows the time responses of the velocity $v_i$ (solid line) and the command $v_i^*$ (dashed line). According to the decision of the priorities, the priority of robot $R_1$ was greater than that of robot $R_2$. It is seen from these figures that a collision was avoided by using only the velocity control. Robot $R_1$ with high priority was accelerated while robot $R_2$ with low priority was decelerated after the cooperative avoidance mode was selected.

**Step 4.** Figure 12 shows the plot of function $f(A_\theta)$ where $l_p$ was given as $l_p = 3.1$. A positive $f(A_\theta)$ means that the collision avoidance condition is satisfied. Positive ranges of $f(A_\theta)$ for $A_\theta > 3.7$ are meaningless because the robots turn more than $2\pi$

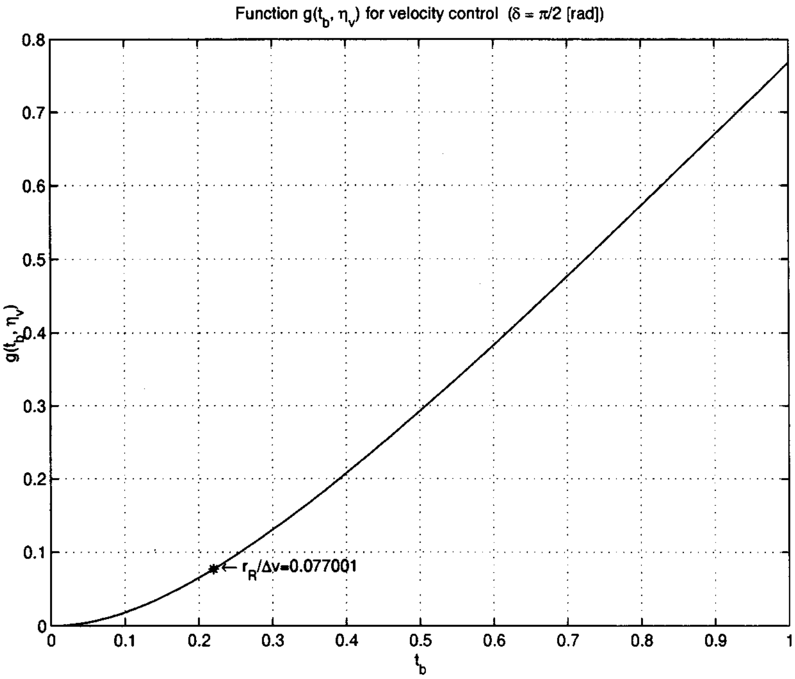


**Figure 9.** Function $g(t_b, \eta_v)$ with $\eta_v = 4.244$.

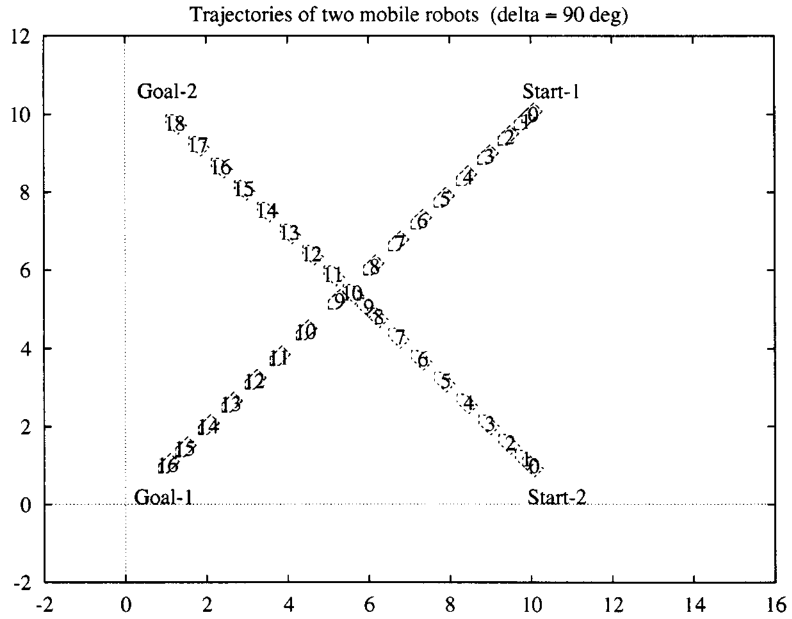


**Figure 10.** Trajectories of two mobile robots for $\delta = \pi/2$ [rad] and $l_p = 3.1$.

during the cooperative avoidance mode. The value of $A_\theta$ was chosen as $A_\theta = 0.45$.

**Step 5.** $k_\theta$ was calculated as 0.7069 [rad] = 40.5 [deg].

Figure 13 shows the trajectories of two mobile robots for $\delta = 0$ [rad] where $k_\theta = 0.7069$ [rad] to hold Eq. (24). Figure 14 shows the time responses of the direction angle $\theta_i$ (solid line) and the command $\theta_i^*$ (dashed line). A collision was avoided by using only the direction control.

To validate Theorems 1 and 2, the following simulation examples are shown. Figure 15 shows the trajectories of two mobile robots with $l_p = 2.4$ which did not satisfy Eq. (34), while Fig. 16 shows the trajectories with $l_p = 3.1$ and $k_\theta = 0.22$ [rad] = 12.6 [deg] which did not satisfy Eq. (24). In both cases, collisions occurred near the predicted crossing point.

### B. $n_r(> 2)$ Mobile Robots

Using the designed navigation parameters in the case of two mobile robots, numerical simulation examples of robots $n_r$ is 3 and 5 are shown. Figure 17 shows the trajectories of three mobile robots with

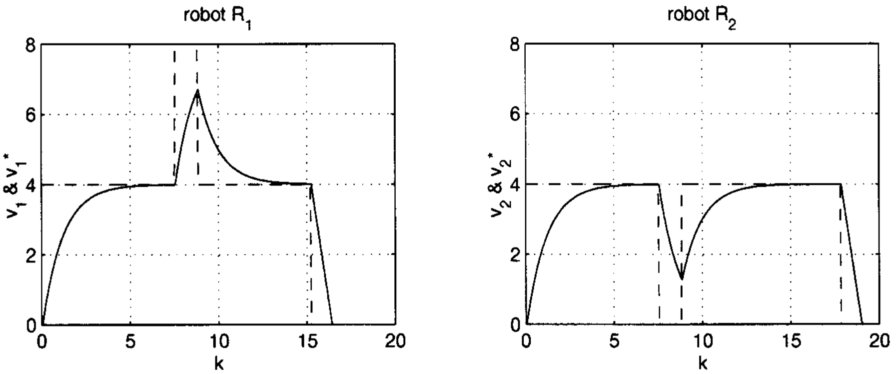


**Figure 11.** Time responses of velocity $v_i$ (solid line) and command $v_i^*$ (dashed line).

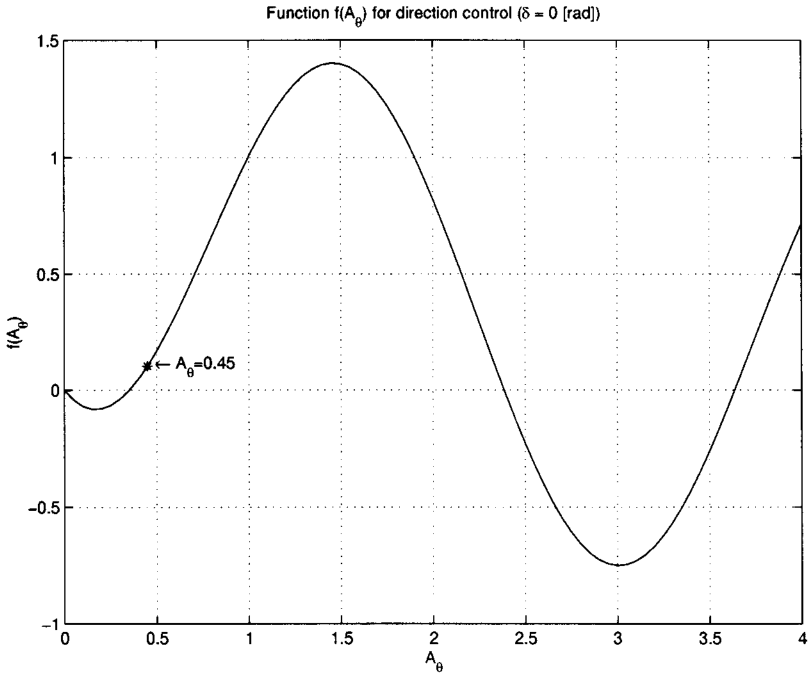


**Figure 12.** Function $f(A_\theta)$ with $l_p = 3.1$.

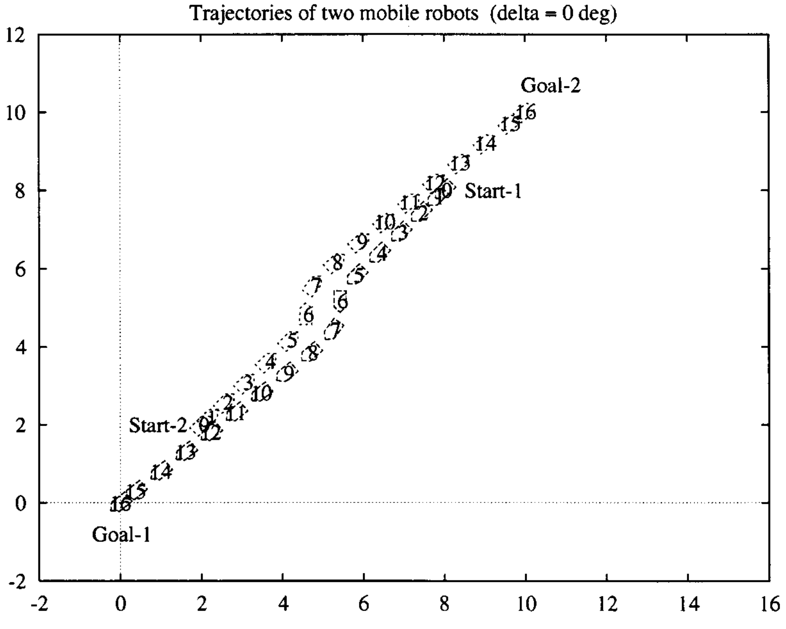


**Figure 13.** Trajectories of two mobile robots for $\delta = 0$ [rad].

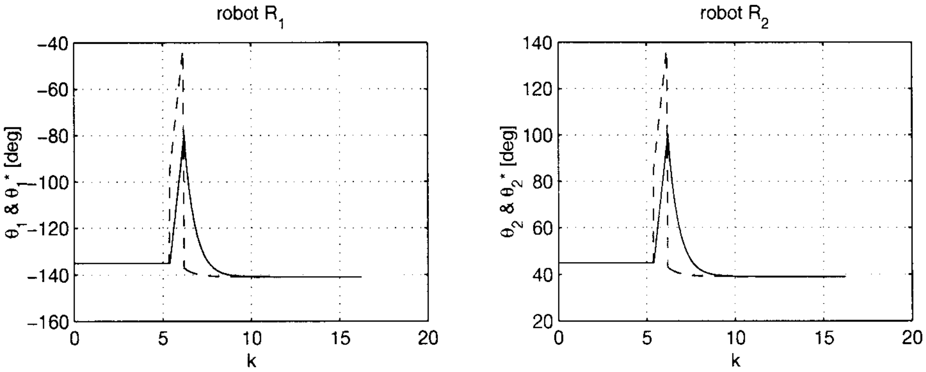


**Figure 14.** Time responses of direction angle $\theta_i$ (solid line) and command $\theta_i^*$ (dashed line).

different starting points and goals. Figure 18 shows the trajectories of five mobile robots. Although some robots had multiple relations with respect to others, their avoidance behaviors were decided by integrating the multiple relations and all the robots were navigated to their own goals without collisions.

Now, we give comments on the collision avoidance conditions, Eqs. (24) and (34), when $n_r > 2$. Since a robot has multiple relations with the rest of robots, the collision avoidance conditions may be violated even if the navigation parameters were chosen according to the procedure given in Section IV B. For example, if robot $R_2$ avoided robot $R_1$ according to the cooperative avoidance behaviors but robot $R_3$ was positioned very close to the area in which robot $R_2$ was moving, the avoidance conditions between robots $R_2$ and $R_3$ may not had held. In the numerical simulation, this phenomenon occurred in proportion as the number of robots was increased. For three mobile robots, collisions were avoided in almost all examples, while collisions were sometimes observed for five mobile robots especially where the robots are positioned *densely*. To improve this, the avoidance behaviors are deter-

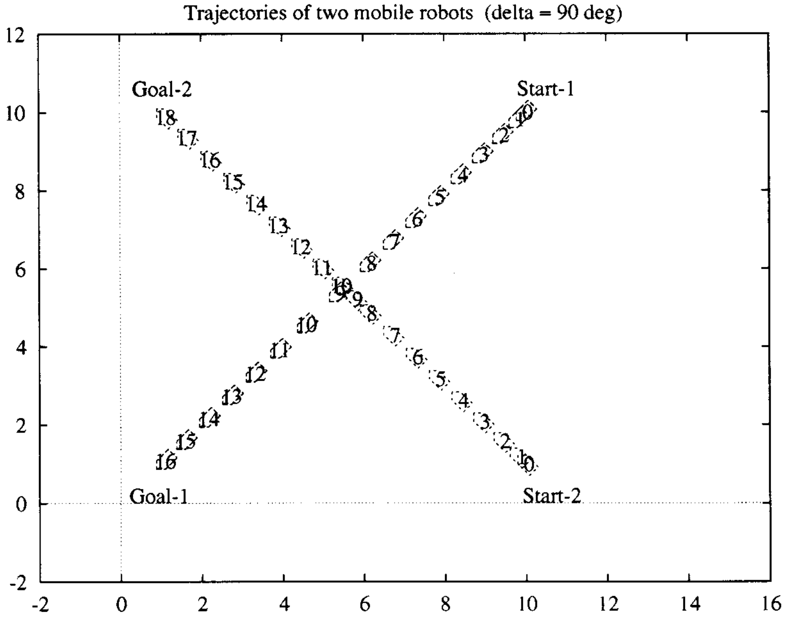


**Figure 15.** Trajectories of two mobile robots for $\delta = \pi/2$ [rad] and $l_p = 2.4$.

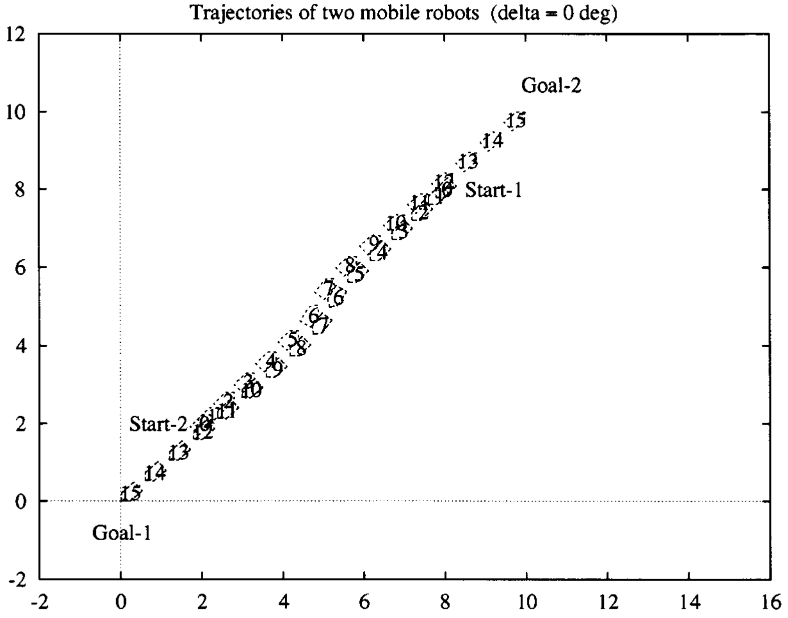


Figure 16. Trajectories of two mobile robots for $\delta = 0$ [rad], $l_p = 3.1$, and $k_\theta = 0.22$ [rad].

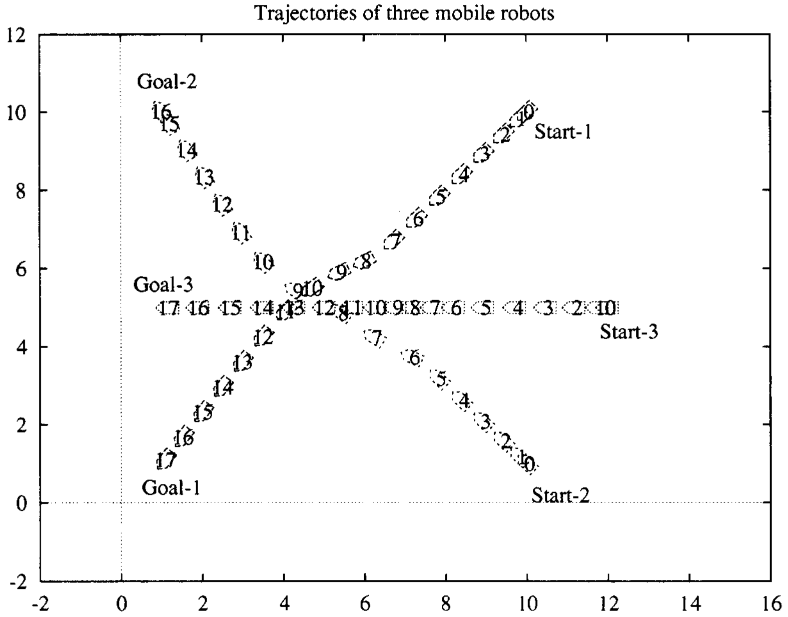


Figure 17. Trajectories of three mobile robots.

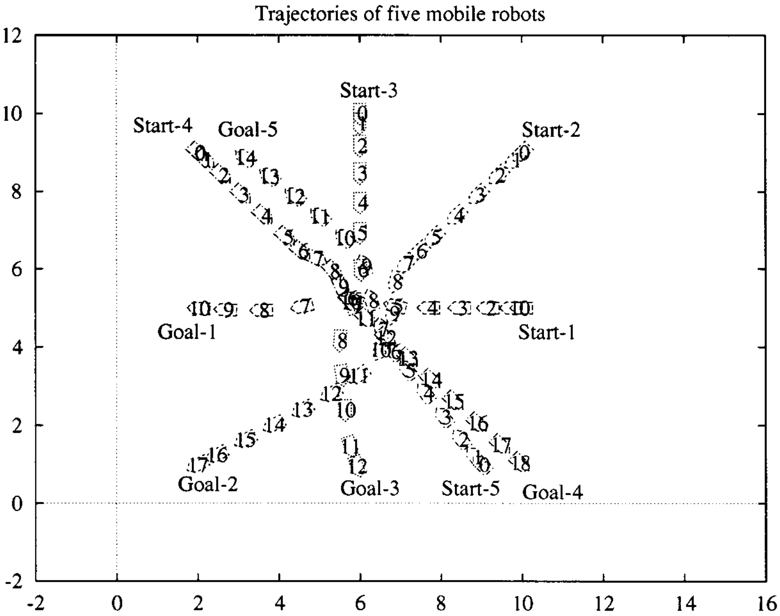


**Figure 18.** Trajectories of five mobile robots.

mined by taking into account not only the crossing angle $\delta$ but also the *robot density* in a region. That is, when increasing the density, it may be a way that robots with lower priority stop or go back to hold the avoidance conditions. This issue will be discussed in a future research.

## VI. NAVIGATION EXPERIMENT USING PIONEER-1

This section shows a navigation experiment using two real mobile robots named *Pioneer-1* to verify the proposed cooperative collision avoidance. Figure 19 shows a photo of Pioneer-1. The radius is 0.15 [m] and the weight is 8 [kg]. Pioneer-1 is a small mobile robot developed by Artificial Intelligence and Grinnell More of Real World Interface Inc.[13] It contains all of basic components for robotics sensing and navigation in a real-world environment, including battery power, drive motors and wheels, position encoders, and ultrasonic sonar transducers, all managed via an onboard MC68HC11-based microprocessor. Pioneer-1 communicates the states of robot and the commands with a client computer through a radio modem.[14]

To perform the proposed cooperative collision avoidance technique experimentally, two vehicles of Pioneer-1 were used as shown in Fig. 20. The states of the robots $(x_i(t), y_i(t), \theta_i(t), v_i(t))$ $(i = 1, 2)$ were in common with the robots through a *memo-link*. A navigation program in which the navigation, cooperative avoidance, and final modes were included was performed by each PC computer. The navigation parameters used in the navigation experiment were given as

$$\begin{aligned} r_R &= 0.15\,[\mathrm{m}], & v_0 &= 1.6\,[\mathrm{m/sec}], \\ \eta_\theta &= 8\,[1/\mathrm{sec}], & \eta_v &= 1.67\,[1/\mathrm{sec}], \\ v_{\max} &= 3.2\,[\mathrm{m/sec}], & v_{\min} &= 0\,[\mathrm{m/sec}], \\ d_p &= 1.2\,[\mathrm{m}], & d_v &= 0.035\,[\mathrm{m}], \\ d_f &= 0.1\,[\mathrm{m}], & k_\theta &= 1. \end{aligned} \tag{48}$$

Substituting these values into equations in Theorem 1, we had $f(A_\theta) = 1.1996$ because of $A_\theta = 0.75$. Similarly in Theorem 2, we had $r_R/\Delta v = 0.09375$, $t_b = 0.415$ [sec], and $g(t_b, \eta_v) = 0.1156$. Thus, the collision avoidance conditions, Eqs. (24) and (34), were satisfied by these values.

Figure 21 shows three experimental results in which the two robots encounter each other with the predicted crossing angle $\delta = 0$, $\pi/4$, and $\pi/2$ [rad]. A sequence of circles means a trajectory of robot and is displayed by 4 [sec]. According to the cooperative avoidance behaviors, collisions were avoided and the robots were navigated to their goals safely in the three cases.

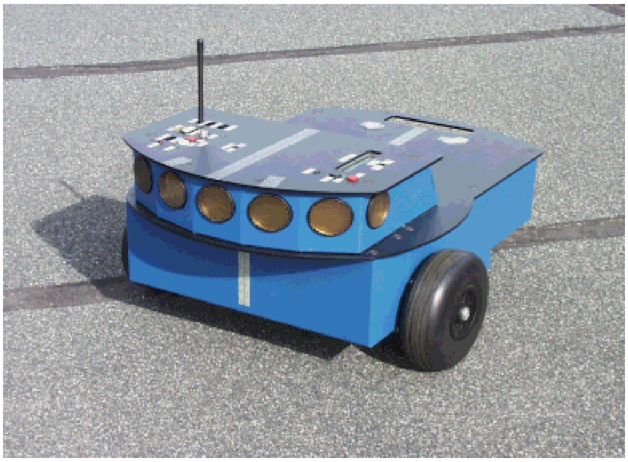


**Figure 19.** Photo of Pioneer-1.

## VII. CONCLUDING REMARKS

This paper has presented a new collision avoidance technique, called cooperative collision avoidance, for multiple mobile robots. The detection of the danger of collision between two mobile robots was discussed with respect to the geometric aspects of their paths. The direction control command and the velocity control command for the cooperative collision avoidance were then proposed. The avoidance technique was extended to cases in which the number of mobile robots is more than two. Furthermore, the conditions for collision avoidance were considered with respect to the navigation parameters and guidelines of designing the navigation parameters were obtained. The effectiveness of the proposed technique was demonstrated by means of numerical simulation and navigation experiments using two real mobile robots named Pioneer-1.

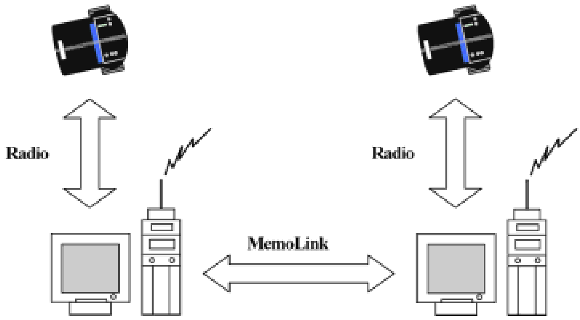
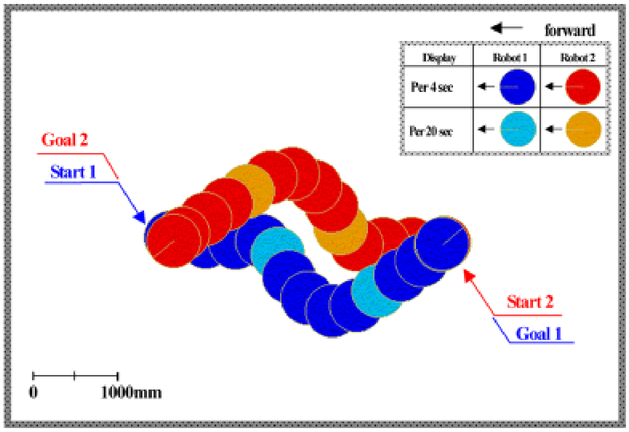
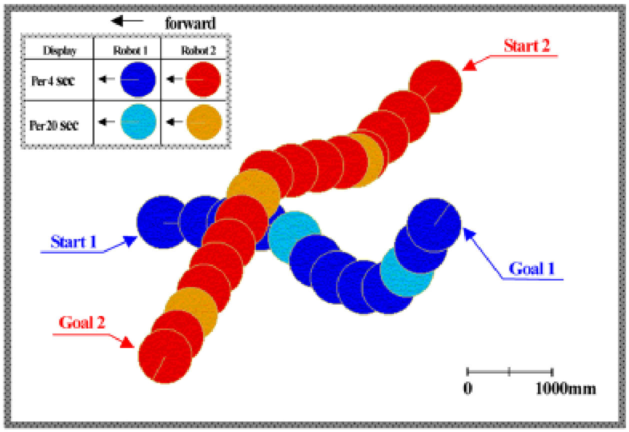


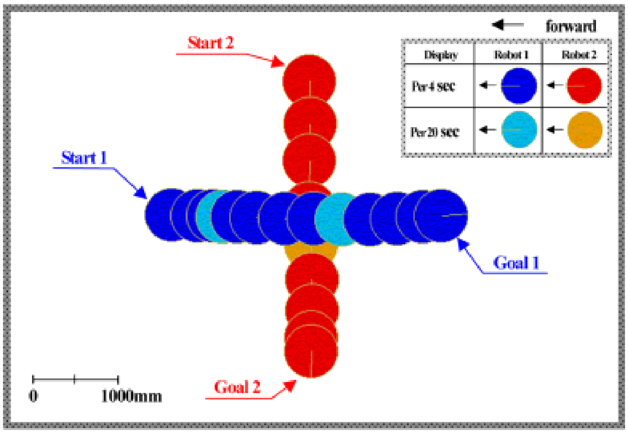
**Figure 20.** Navigation experimental system using two vehicles of Pioneer-1.



(a) =0 [rad]



(b) = /4 [rad]



(c) = /2 [rad]

**Figure 21.** Trajectories of two vehicles of Pioneer-1.

This paper gave an insight into the navigation parameters of multiple mobile robots from the theoretical points of view. The obtained results are also helpful for the learning techniques to choose the initial values of the parameters. As further subjects to research, the avoidance behaviors should be improved to hold the avoidance conditions when increasing the number of robots. In this paper, it was supposed that there was no obstacle in the environment. The proposed technique should be applied to more complicated environments in which static and moving obstacles are scattered.

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