7.46. **Riding a Loop-the-Loop.** A car in an amusement park ride rolls without friction around the track shown in the figure. It starts from rest at point $A$ at a height $h$ above the bottom of the loop. Treat the car as a particle. (a) What is the minimum value of $h$ (in terms of $R$) such that the car moves around the loop without falling off at the top (point $B$)? (b) If $h = 3.50R$ and $R = 20.0$ m, compute the speed, radial acceleration, and tangential acceleration of the passengers when the car is at point $C$, which is at the end of a horizontal diameter. Show these acceleration components in a diagram, approximately to scale.

**Identify:** Apply $K_i + U_i + W_{nc} = K_2 + U_2$ to relate $h$ and $v_h$. Apply $\sum F = ma$ at point $B$ to find the minimum speed required at $B$ for the car not to fall off the track.

**Set up:** At $B$, $a = \frac{v_h^2}{R}$, downward. The minimum speed is when $n \to 0$ and $mg = \frac{mv_h^2}{R}$. The minimum speed required is $v = \sqrt{gR}$. $K_i = 0$ and $W_{nc} = 0$.

**Execute:**

(a) $K_A + U_A = K_B + U_B$ applied to points $A$ and $B$ gives $U_A - U_B = \frac{1}{2}mv_h^2$. The speed at the top must be at least $\sqrt{gR}$. Thus, $mg(h - 2R) > \frac{1}{2}mgR$, or $h > \frac{5}{2}R$.

(b) Apply $K_A + U_A = K_C + U_C$ to points $A$ and $C$. $U_A - U_C = (2.50)Rmg = K_C$, so $v_C = \sqrt{(5.00)gR} = \sqrt{(5.00)(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 31.3 \text{ m/s}$.

The radial acceleration is $a_{rad} = \frac{v_C^2}{R} = 49.0 \text{ m/s}^2$. The tangential direction is down, the normal force at point $C$ is horizontal, there is no friction, so the only downward force is gravity, and $a_{tan} = g = 9.80 \text{ m/s}^2$.

**Evaluate:** If $h > \frac{5}{2}R$, then the downward acceleration at $B$ due to the circular motion is greater than $g$ and the track must exert a downward normal force $n$. $n$ increases as $h$ increases and hence $v_h$ increases.