8.49. **Pluto and Charon.** Pluto’s diameter is approximately 2370 km, and the diameter of its satellite Charon is 1250 km. Although the distance varies, they are often about 19,700 km apart, center-to-center. Assuming that both Pluto and Charon have the same composition and hence the same average density, find the location of the center of mass of this system relative to the center of Pluto.

**Identify:** The location of the center of mass is given by \( \overline{r}_{CM} \equiv \frac{\sum m_j \overline{r}_j}{\sum m_j} \). The mass can be expressed in terms of the diameter. Each object can be replaced by a point mass at its center.

**Set Up:** Use coordinates with the origin at the center of Pluto and the +x direction toward Charon, so \( x_p = 0 \) \( x_c = 19,700 \) km \( m = \rho V = \rho \frac{4}{3} \pi r^3 = \frac{1}{6} \rho \pi d^3 \).

\[
x_m = \frac{m_p x_p + m_c x_c}{m_p + m_c} = \left( \frac{m_c}{m_p + m_c} \right) x_c = \left( \frac{\frac{1}{6} \rho \pi d_p^3}{\frac{1}{6} \rho \pi d_p^3 + \frac{1}{6} \rho \pi d_c^3} \right) x_c = \left( \frac{d_c^3}{d_p^3 + d_c^3} \right) x_c.
\]

**Execute:**

\[
x_m = \left( \frac{[1250 \text{ km}]^3}{[2370 \text{ km}]^3 + [1250 \text{ km}]^3} \right) (19,700 \text{ km}) = 2.52 \times 10^3 \text{ km}
\]

The center of mass of the system is 2.52 \times 10^3 \text{ km} from the center of Pluto.

**Evaluate:** The center of mass is beyond the radius of Pluto by more than twice its value. The center of mass is closer to Pluto because Pluto has more mass than Charon.