13.78. A uniform beam is suspended horizontally by two identical vertical springs that are attached between the ceiling and end of the beam. The beam has mass 225 kg, and a 175-kg sack of gravel sits on the middle of it. The beam is oscillating in SHM, with and amplitude of 40.0 cm and a frequency of 0.600 cycles/s. (a) The sack of gravel falls off the beam when the beam has its maximum upward displacement. What are the frequency and amplitude of the subsequent SHM of the beam? (b) If the gravel instead falls off when the beam has its maximum speed, what are the frequency and amplitude of the subsequent SHM of the beam?

Identify: \[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]. Use energy considerations to find the new amplitude.

Set Up: This is the effective force constant of the two springs.

(a) After the gravel sack falls off, the remaining mass attached to the springs is 225 kg. The force constant of the springs is unaffected, so \( f = 0.800 \text{ Hz} \). To find the new amplitude use energy considerations to find the distance downward that the beam travels after the gravel falls off. Before the sack falls off, the amount \( x_0 \) that the spring is stretched at equilibrium is given by \( mg - kx_0 \), so \( x_0 = mg/k = (400 \text{ kg})(9.80 \text{ m/s}^2)/(5685 \text{ N/m}) = 0.6895 \text{ m} \). The maximum upward displacement of the beam is \( A = 0.400 \text{ m} \) above this point, so at this point the spring is stretched 0.2895 m. With the new mass, the mass 225 kg of the beam alone, at equilibrium the spring is stretched \( mg/k = (225 \text{ kg})(9.80 \text{ m/s}^2)/(5685 \text{ N/m}) = 0.6895 \text{ m} \). The new amplitude is therefore 0.3879 m – 0.2895 m = 0.098 m. The beam moves 0.098 m above and below the new equilibrium position. Energy calculations show that \( v = 0 \) when the beam is 0.098 m above and below the equilibrium point.

(b) The remaining mass and the spring constant is the same in part (a), so the new frequency is again 0.800 Hz. The sack falls off when the spring is stretched 0.6895 m. And the speed of the beam at this point is \( v = A\sqrt{k/m} = (0.400 \text{ m})\sqrt{(5685 \text{ N/m})(400 \text{ kg})} = 1.508 \text{ m/s} \). Take \( y = 0 \) at this point. The total energy of the beam at this point, just after the sack falls off, is \( E = K + U_{el} + U_{grav} = \frac{1}{2}(225 \text{ kg})(1.508 \text{ m/s}^2) + \frac{1}{2}(5695 \text{ N/m})(0.6895 \text{ m})^2 + 0 = 1608 \text{ J} \). Let this be point 1. Let point 2 be where the beam has moved upward a distance \( d \) and where \( v = 0 \). \[ E_2 = \frac{1}{2}k(0.6985 \text{ m} - d)^2 + mgd \]. Energy calculations show that \( v \) is also zero when the beam is 0.426 m below the equilibrium position.

Evaluate: The new frequency is independent of the point in the motion at which the bag falls off. The new amplitude is smaller than the original amplitude when the sack falls off at the maximum upward displacement of the beam. The new amplitude is larger than the original amplitude when the sack falls off when the beam has maximum speed.