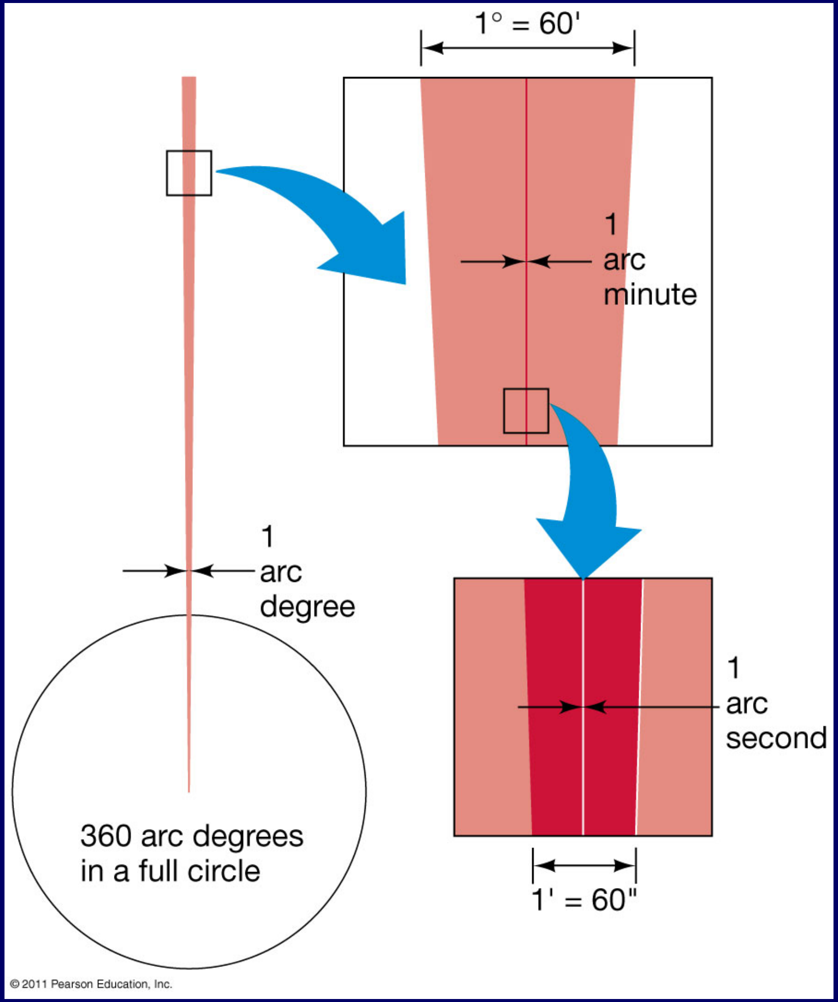
The Family of Stars

- Distance
- Luminosity
- Surface temperature
- Composition
- Radius
- Mass

Angular Measure

- Full circle contains 360° (degrees)
- Each degree contains 60′ (arc-minutes)
- Each arc-minute contains 60″ (arc-seconds)
- Angular size of an object depends on its actual size and distance from viewer



The Measurement of Distance

Triangulation:
Measure the baseline distance b and angles θ and φ, then calculate the distance d. If $\varphi = 90°$, then

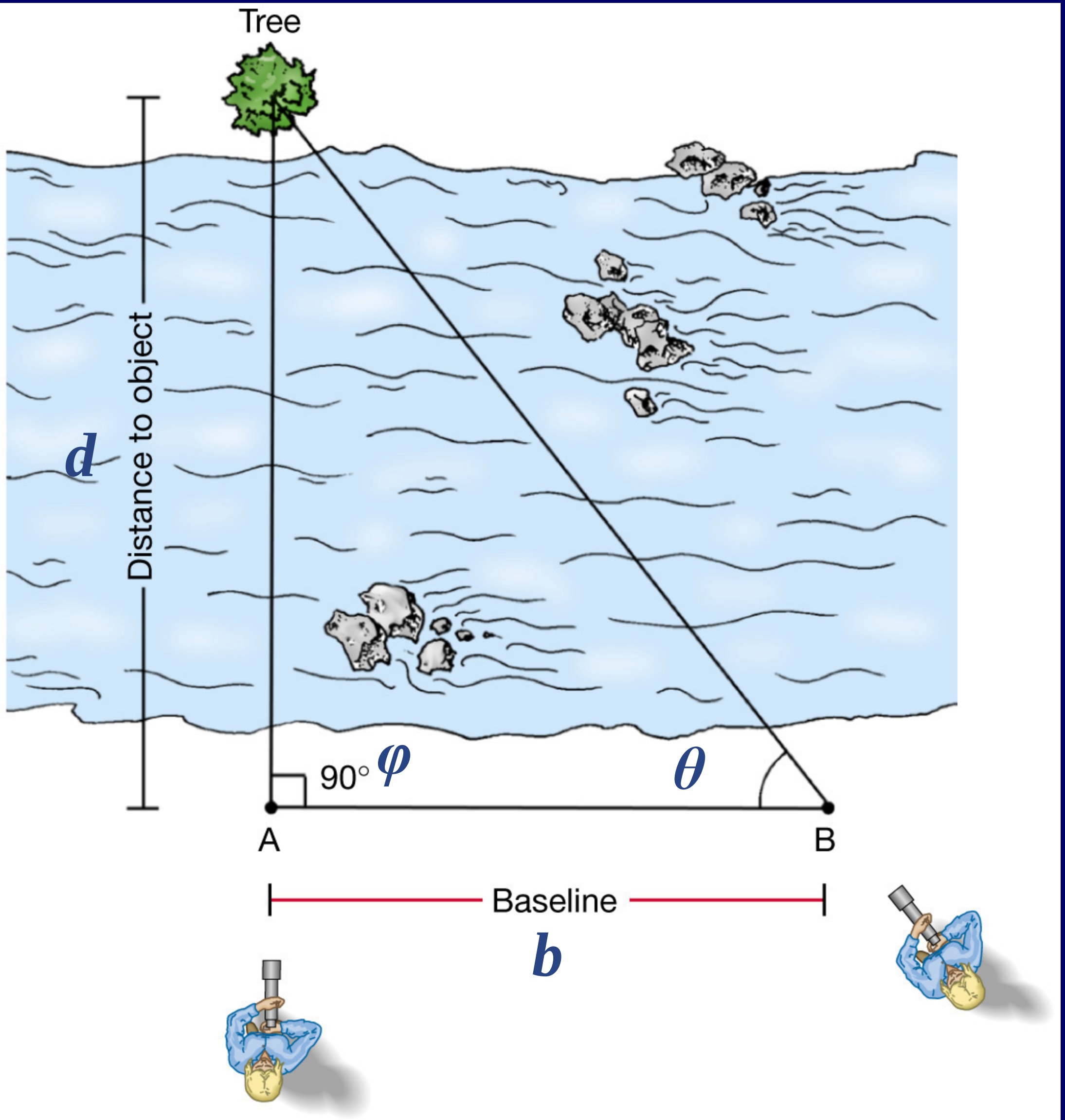
$$d = b/\tan\theta$$

If θ is small

$\tan\theta \approx \theta$ (in radians),

then

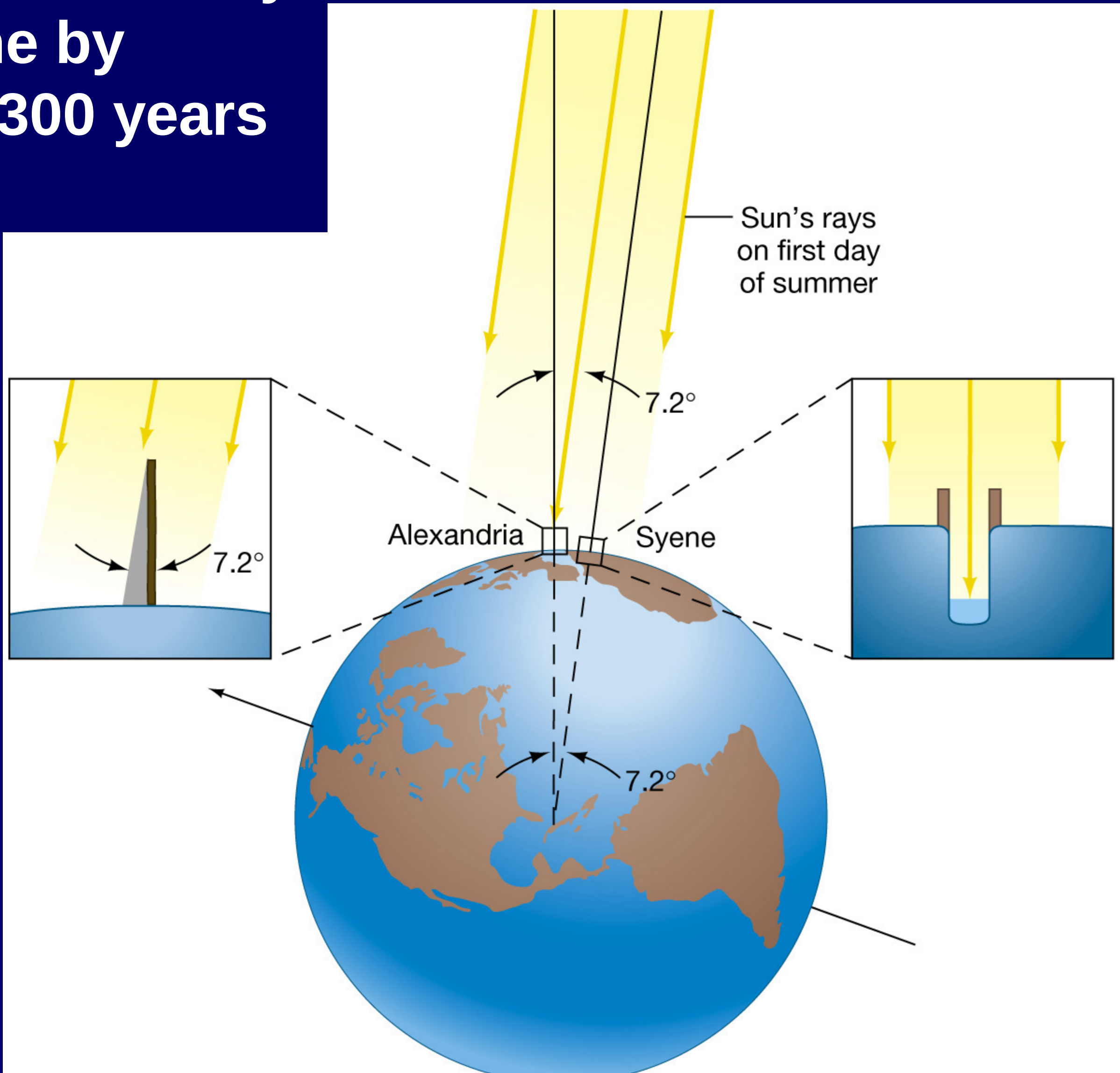
$$d \approx b/\theta$$



The Measurement of Distance

Measuring the Earth's radius by triangulation was done by Eratosthenes about 2300 years ago.

He noticed that when the Sun was directly overhead in Syene, it was at an angle in Alexandria. By measuring the angle θ and the distance b between the cities he calculated the radius $r = b/\theta$.



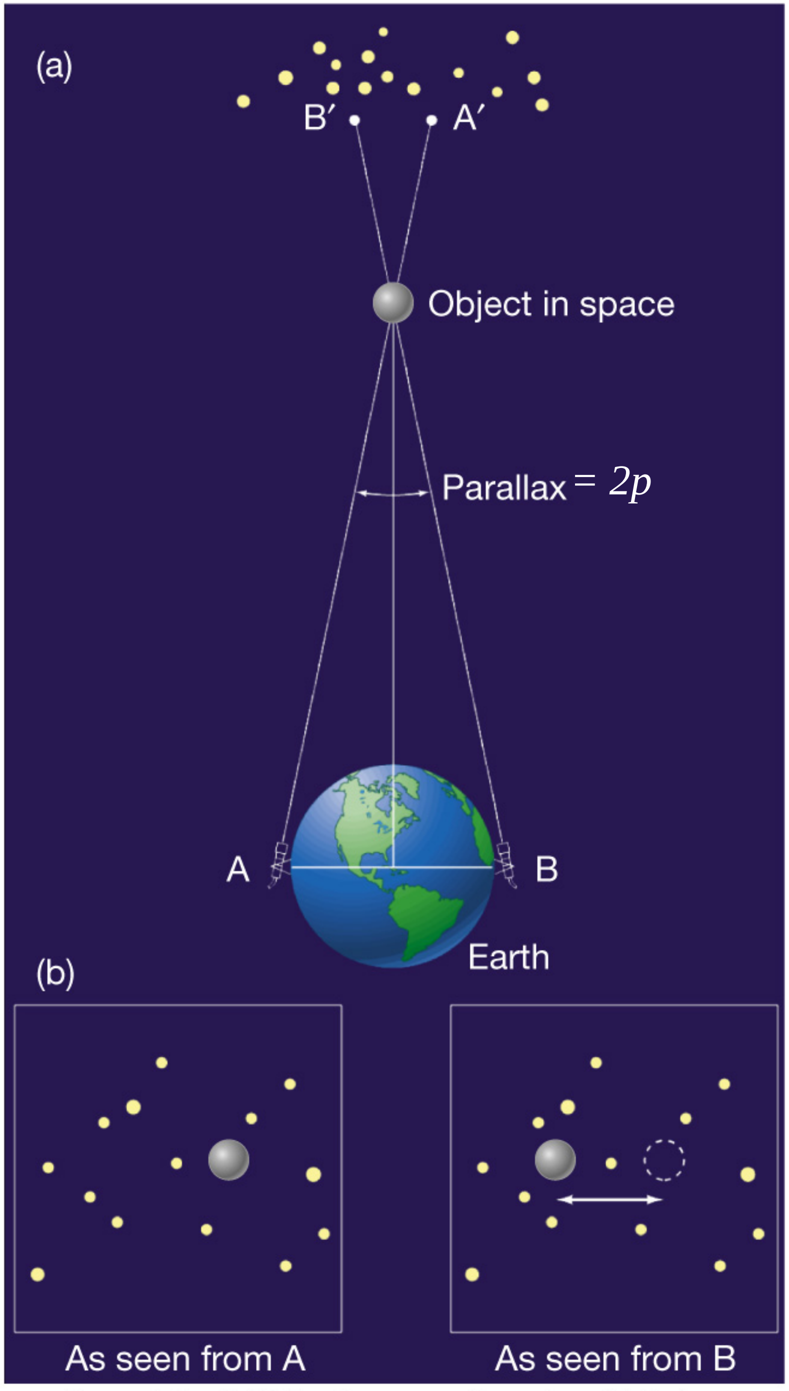
The Measurement of Distance

Trigonometric parallax: is a form of triangulation, but we look at the apparent displacement of an object against the fixed stars from two vantage points

In this example, we observe on opposite sides of the Earth (diameter of the Earth is the baseline).

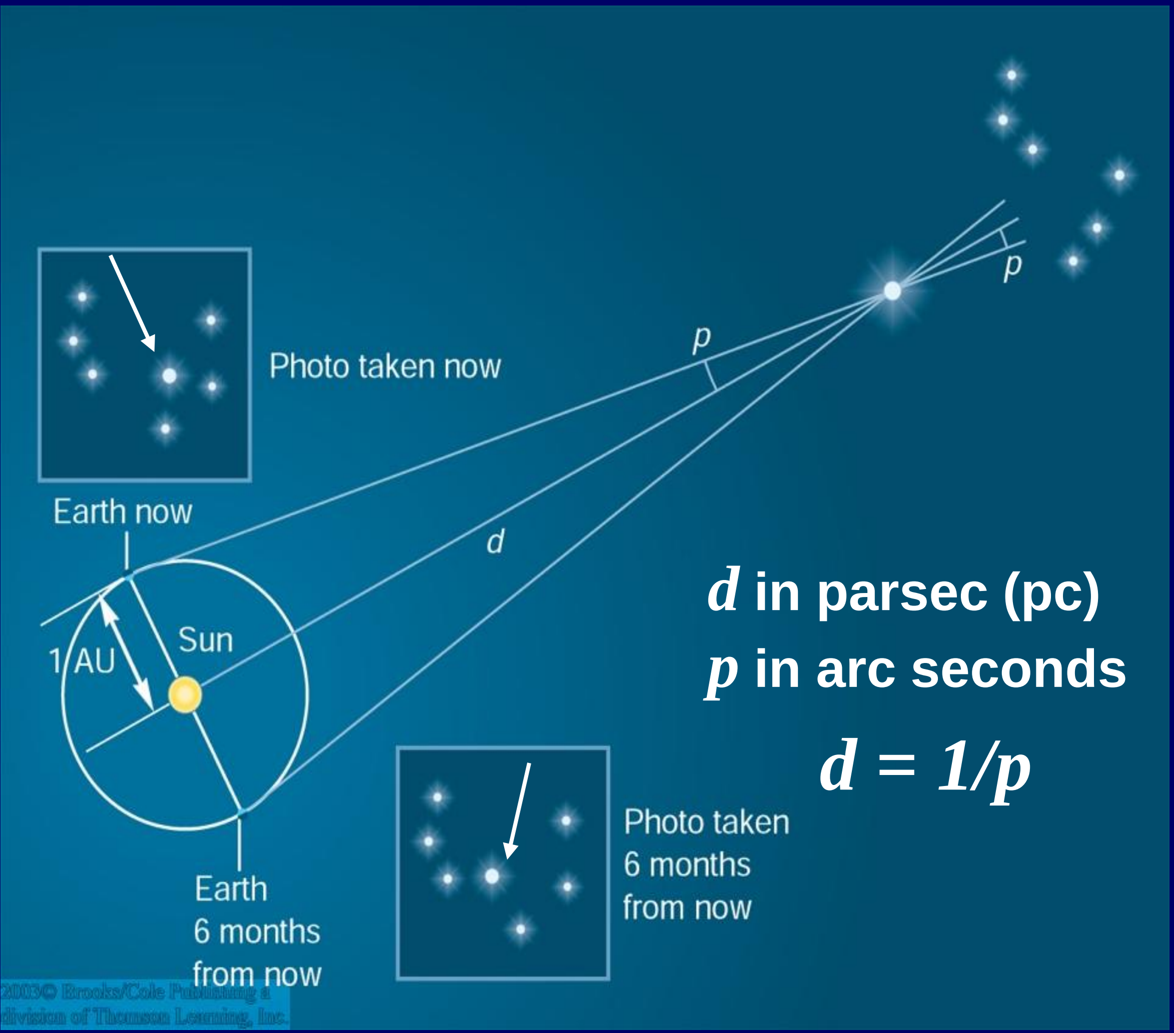
Then, $d \approx r_{\oplus}/p$

Stars are too far away for this to work because the angles are too small to measure.



Distances to Stars

Using the Earth's orbit as a baseline:



d in parsec (pc)

p in arc seconds

$$d = 1/p$$

A star appears slightly shifted from different positions of the Earth on its orbit. The farther away the star is (larger d), the smaller the parallax angle p.

A parsec is based on a b = 1 AU and p = 1 sec^{-1}

1 pc = 3.26 LY

Trigonometric Parallax

Example:

- **The nearest star, α Centauri, has a parallax of $p = 0.76$ arc seconds for a baseline $b = 1$ AU**

$$d = 1/p = 1/0.76 = 1.3 \text{ pc} = 1.3 \times 3.26 = 4.3 \text{ LY}$$

- **With ground-based telescopes, we can measure parallaxes $p \geq 0.02$ arc sec**
  $\Rightarrow d \leq 50$ pc

- **For this reason, with ground based telescopes we are limited to stars no farther away than 50 pc (~10,000 stars).**

- **The Hipparcos satellite enabled us to increase the number to ~120,000 stars out to ~110 pc**

The Solar Neighborhood

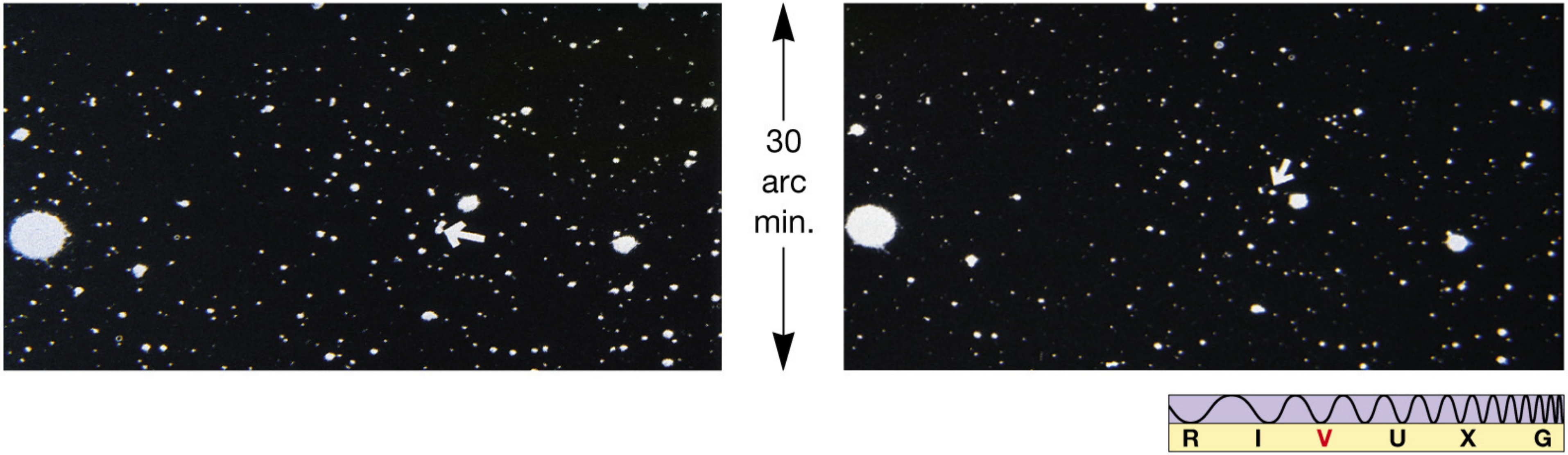
The nearest star to the Sun, Proxima Centauri, is a member of a 3-star system: the α Centauri complex

Model of distances:

- If the Sun is a marble, then the Earth is a grain of sand orbiting 1 m away
- The nearest star is another marble 270 km (~170 miles) away
- The solar system extends about 50 m from the Sun; the rest of the distance to the nearest star is basically empty

The Solar Neighborhood

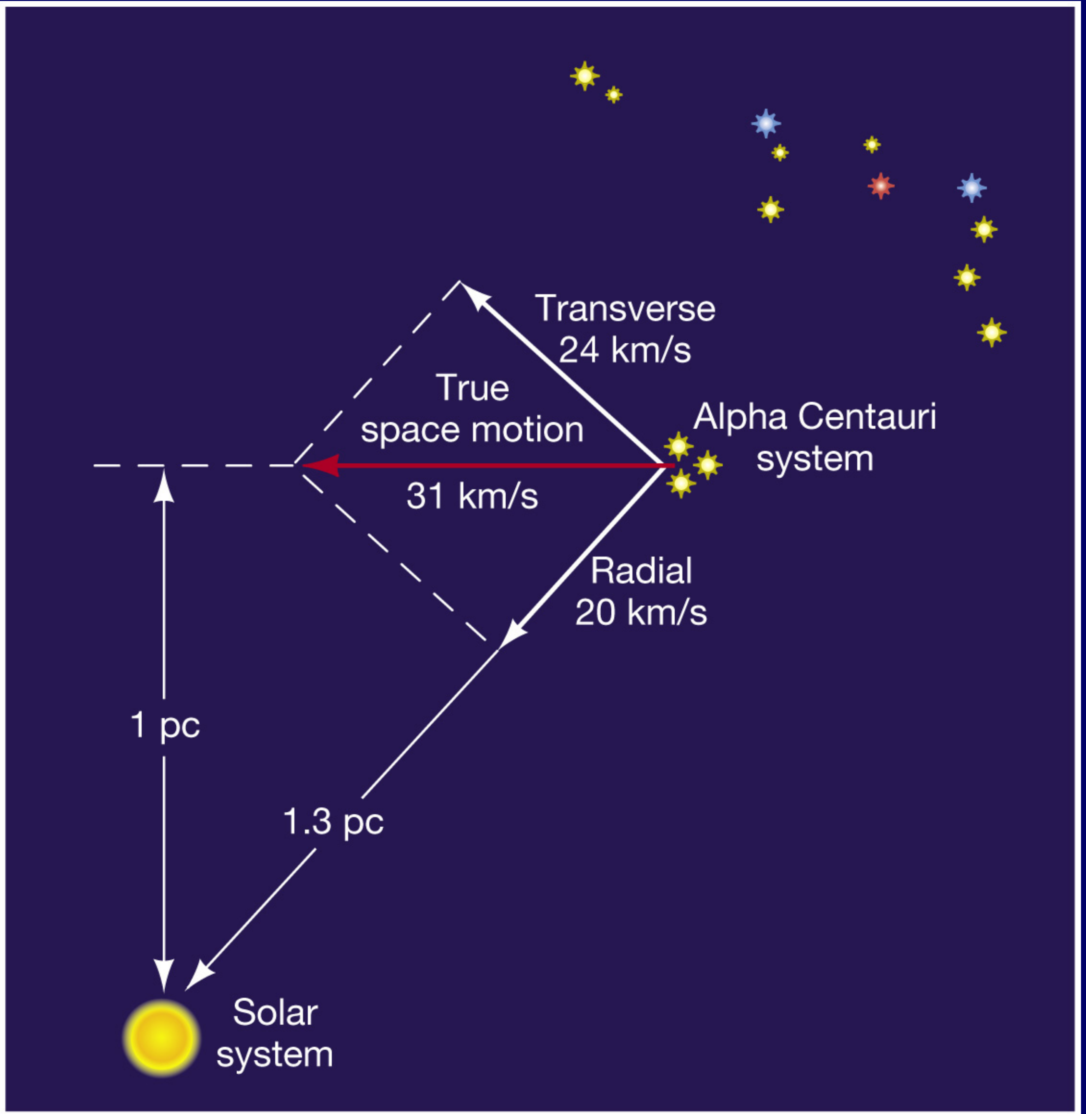
- The next nearest neighbor: Barnard's Star
- Barnard's Star has the largest proper motion of any star after correcting for parallax.
- Proper motion is the actual transverse movement of the star in space.
- These pictures were taken 22 years apart



The Solar Neighborhood

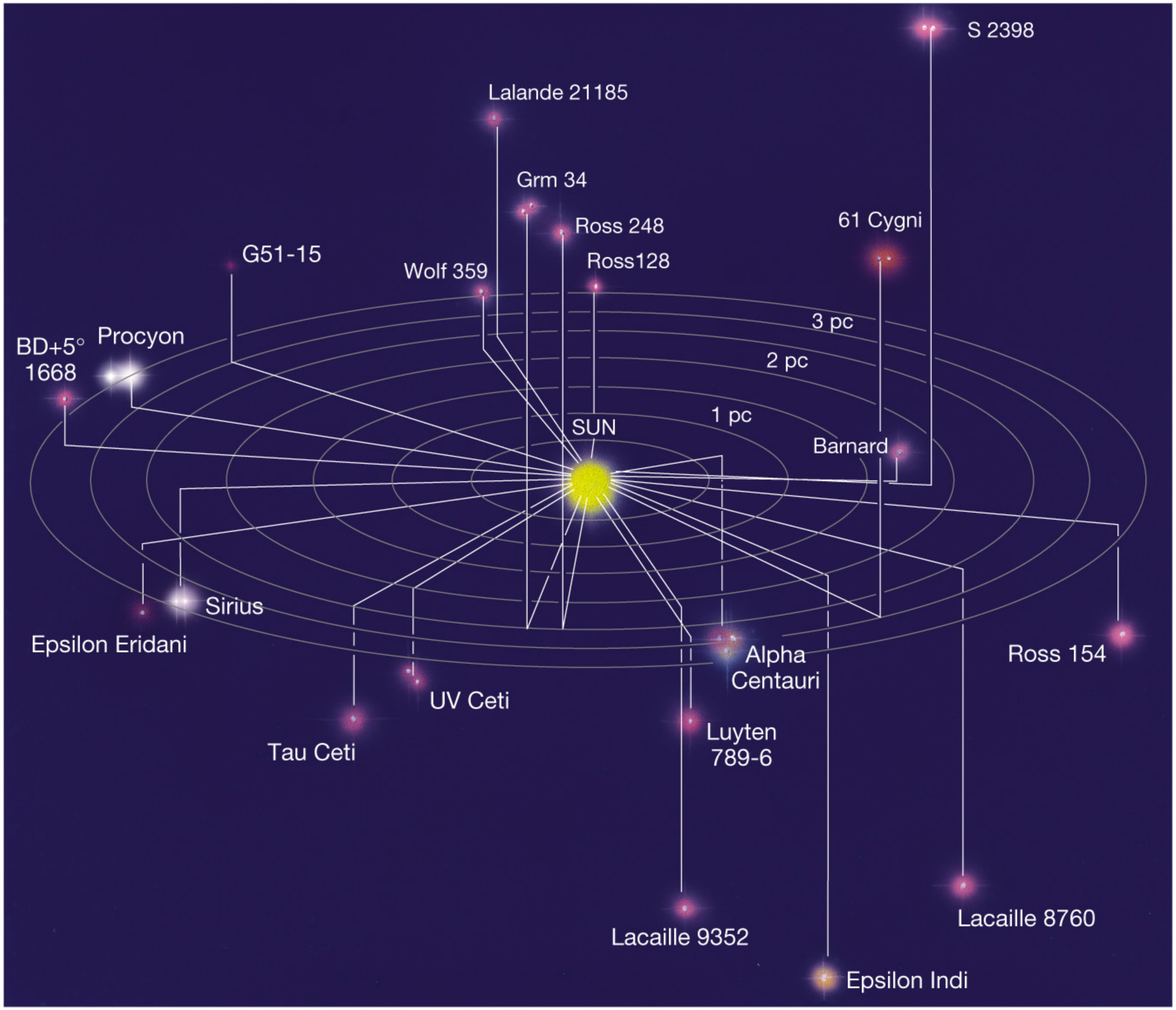
The actual motion of the α Centauri complex:

- The transverse speed is determined from the proper motion measured over many years.
- The radial speed is measured from a Doppler shift $v_r = c\, \Delta\lambda/\lambda_0$.
- The true space motion is the vector sum of the transverse speed and the radial speed.



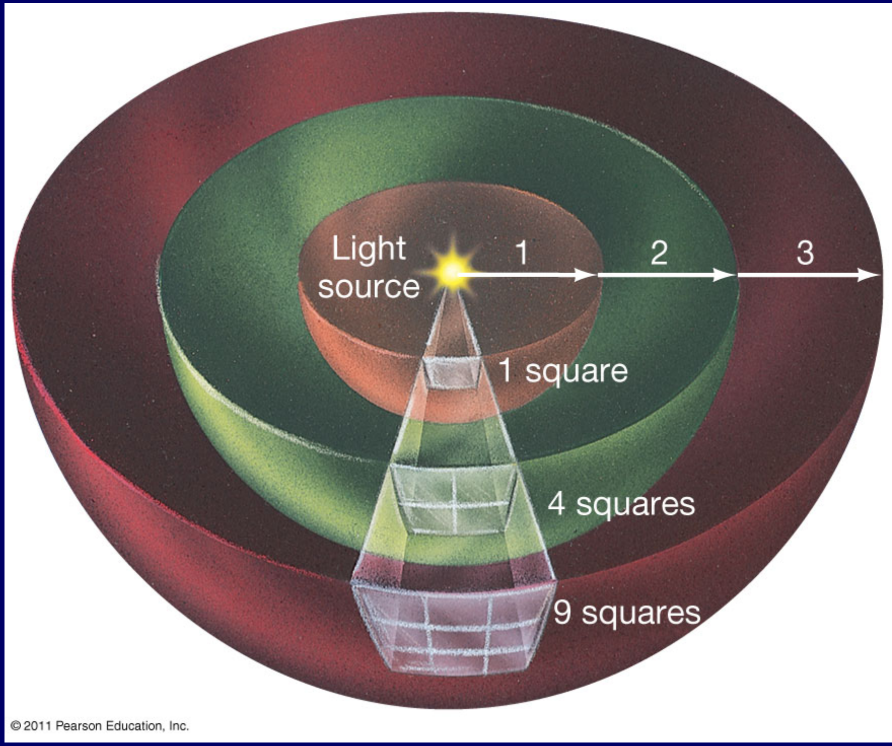
The Solar Neighborhood

The 30 closest stars to the Sun



Luminosity Relative to Apparent Brightness

- Luminosity is the flux of radiation energy from the star (its intrinsic brightness)
- The more distant a light source, the fainter it appears.
- The same amount of light falls on a smaller area at a distance of 1 unit than at a distance of 2 units $\Rightarrow$ smaller apparent brightness.



- Area increases as distance squared $\Rightarrow$ apparent brightness decreases as the inverse of distance squared

Intrinsic Brightness Relative to Apparent Brightness

In an equation, the flux b (apparent brightness) received from the source is proportional to its intrinsic brightness or luminosity L and inversely proportional to the square of the distance d

$$\boxed{b \propto \frac{L}{d^2}}$$

Both stars may appear equally bright ($b_A = b_B$), whereas star A is intrinsically much brighter than star B ($L_A > L_B$) because $d_A > d_B$

The Magnitude Scale

First introduced by Hipparchus (160-127 BC):

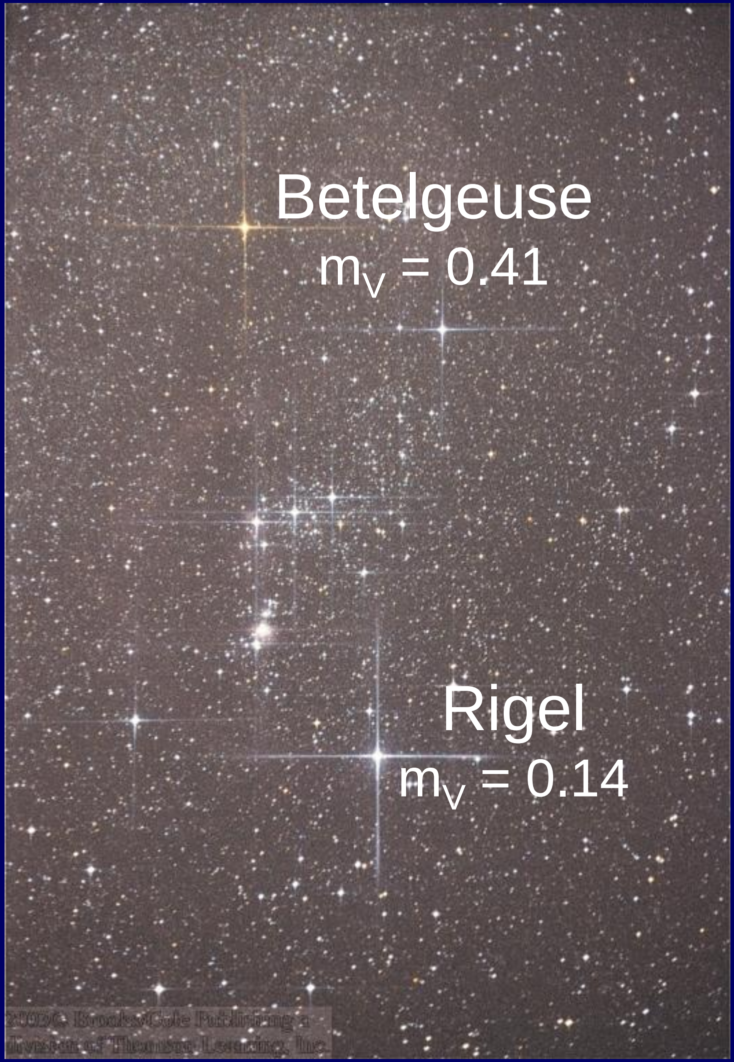
- **Brightest stars: 1st magnitude ($m_V = 1$)**
- **Faintest stars (unaided eye): 6th magnitude ($m_V = 6$)**

In the 19th century, it was found that:

- **1 m_V stars appear ~100 times brighter than 6 m_V stars and the scale is logarithmic**
- **It was then defined that 1 m_V difference gives a factor of 2.512 in apparent brightness (larger magnitude = fainter object!) such that a difference of 5 m_V is exactly 100 times difference in brightness**

Example:
Betelgeuse and Rigel

Magnitude Difference	Intensity Ratio
1	2.512
2	$2.512 * 2.512 = (2.512)^2 = 6.31$
3	$2.512 * 2.512 * 2.512 = (2.512)^3 = 15.85$
4	$(2.512)^4 = 39.82$
5	$(2.512)^5 = 100$



For a magnitude difference of 0.41 – 0.14 = 0.27, we find an intensity ratio of $(2.512)^{0.27} = 1.28$

The Magnitude Scale

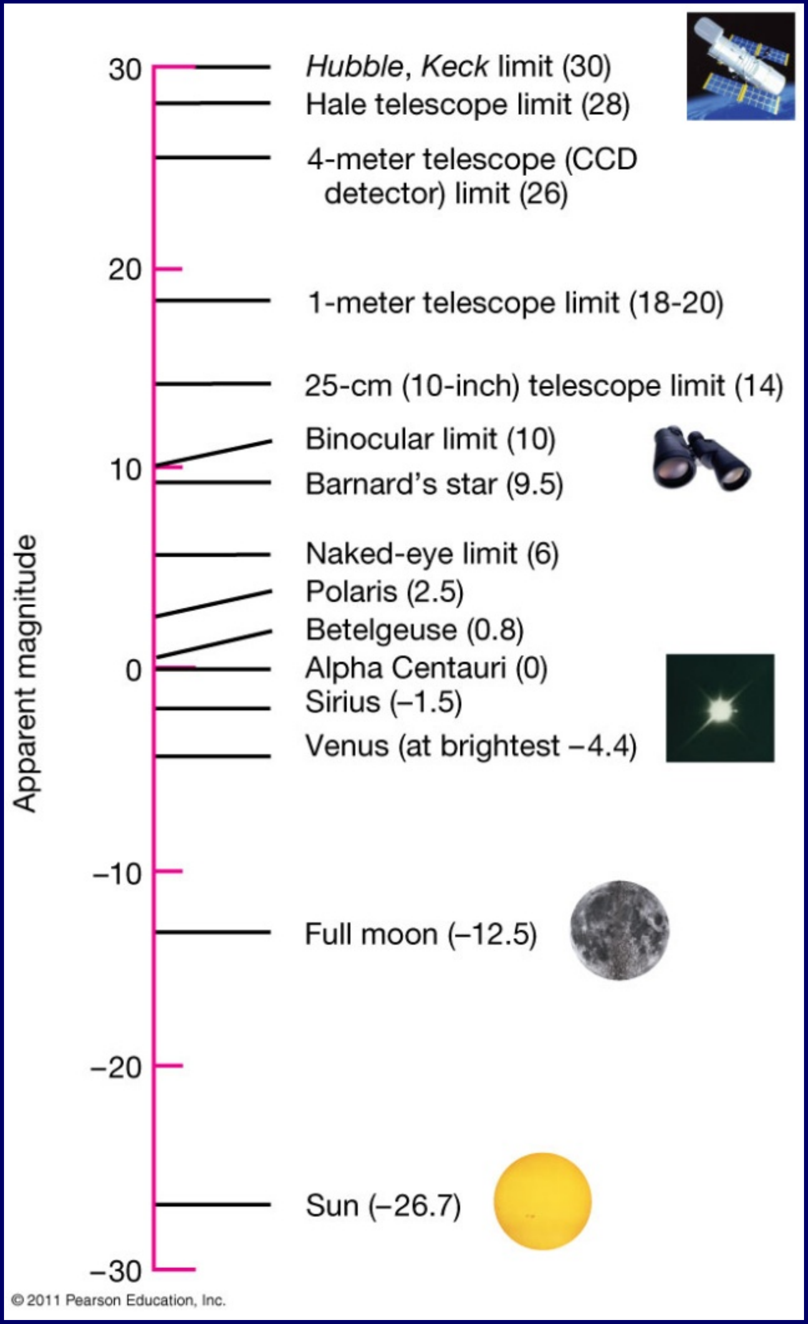
The magnitude scale system can be extended towards negative numbers (very bright) and numbers > 6 (faint objects):

Sirius (brightest star in the sky): m_v = -1.42

Venus (maximum): m_v = -4.4

Full Moon: m_v = -12.5

Sun: m_v = -26.7

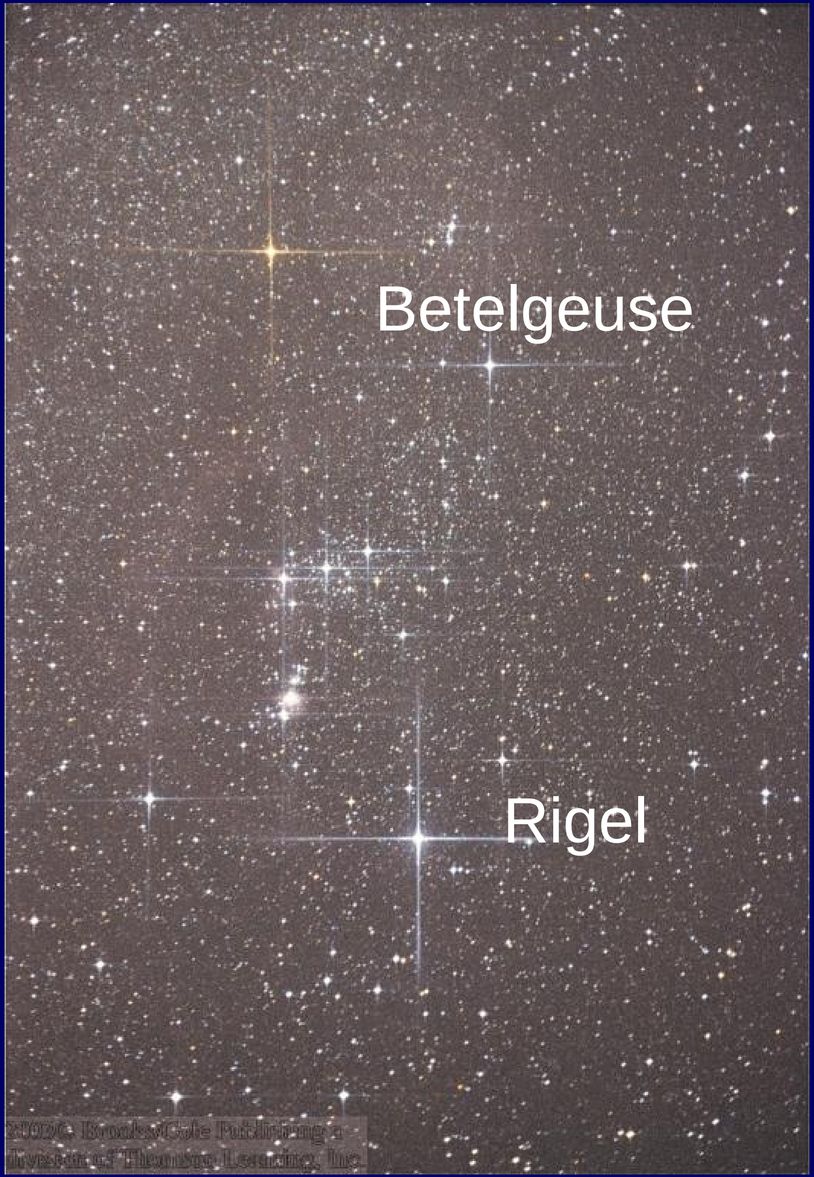


Distance and Intrinsic Brightness

Earlier we determined that Rigel appears 1.28 times brighter than Betelgeuse,

But Rigel is 1.6 times further away than Betelgeuse

In fact, Rigel is actually (intrinsically) $1.28*(1.6)^2 =$ 3.3 times brighter than Betelgeuse.



Absolute Visual Magnitude

- To characterize a star's intrinsic brightness (luminosity L), we define absolute visual magnitude M_V as the apparent visual magnitude (brightness) that a star would have if it were at a distance of 10 pc.
- Both M_V and L state a star's intrinsic brightness.
- M_V is most useful for comparing stars
- L is expressed in physical units of power (*e.g.* Watts)

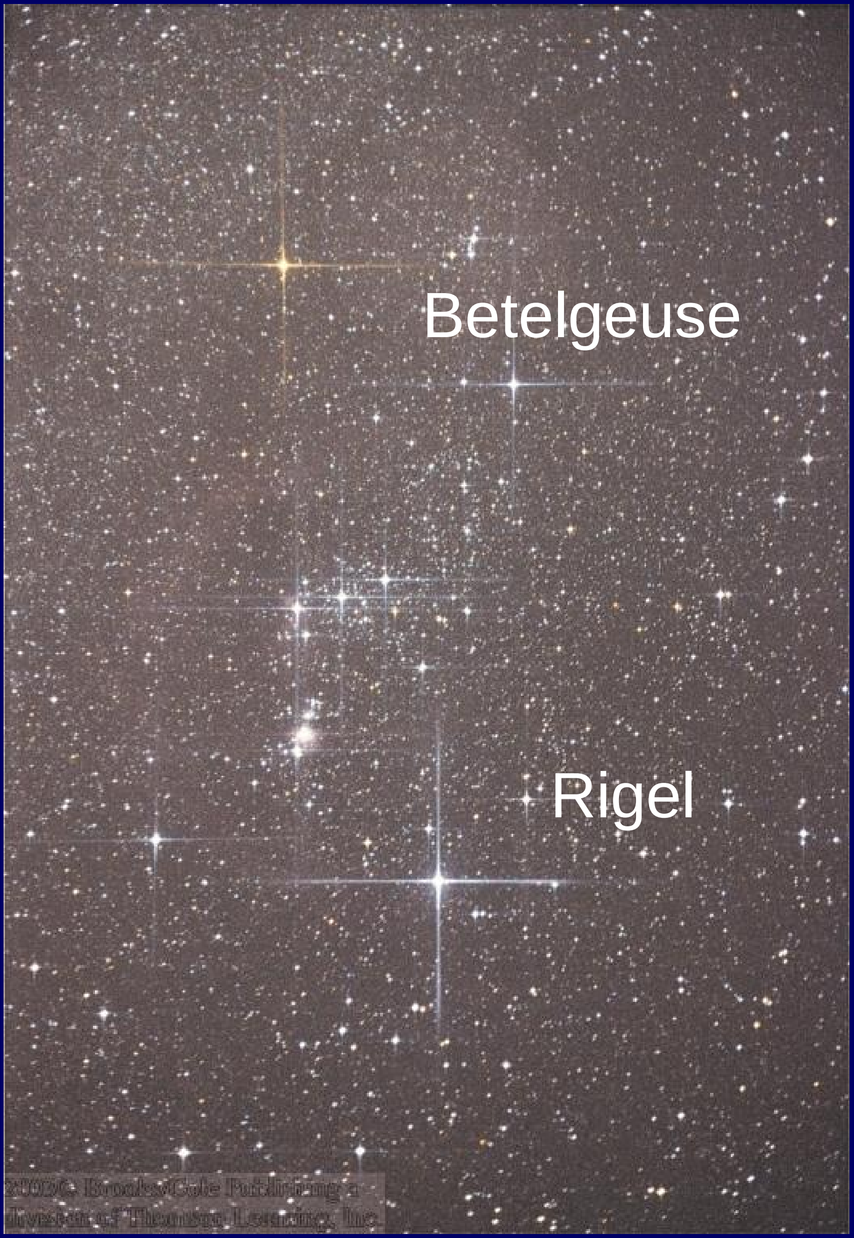
Absolute Visual Magnitude

Back to our example of Betelgeuse and Rigel:

	Betelgeuse	Rigel
m_V	0.41	0.14
d	152 pc	244 pc
M_V	-5.5	-6.8

Difference in absolute magnitudes: 6.8 – 5.5 = 1.3

$\Rightarrow$ Luminosity ratio = $(2.512)^{1.3}$ = 3.3
as we saw before



The Distance Modulus

If we know a star's absolute magnitude, we can infer its distance by comparing absolute and apparent magnitudes

Distance Modulus

$$= m_V - M_V$$

$$= -5 + 5\log_{10}(d\ [\mathrm{pc}])$$

Distance in units of parsec

Equivalent:

$$d = 10^{(m_V - M_V + 5)/5}\ \mathrm{pc}$$

This formula is important for finding distances to stars using "standard candle" methods

Luminosity and Apparent Brightness

If we know a star's brightness and its distance from us, we can calculate its absolute magnitude.

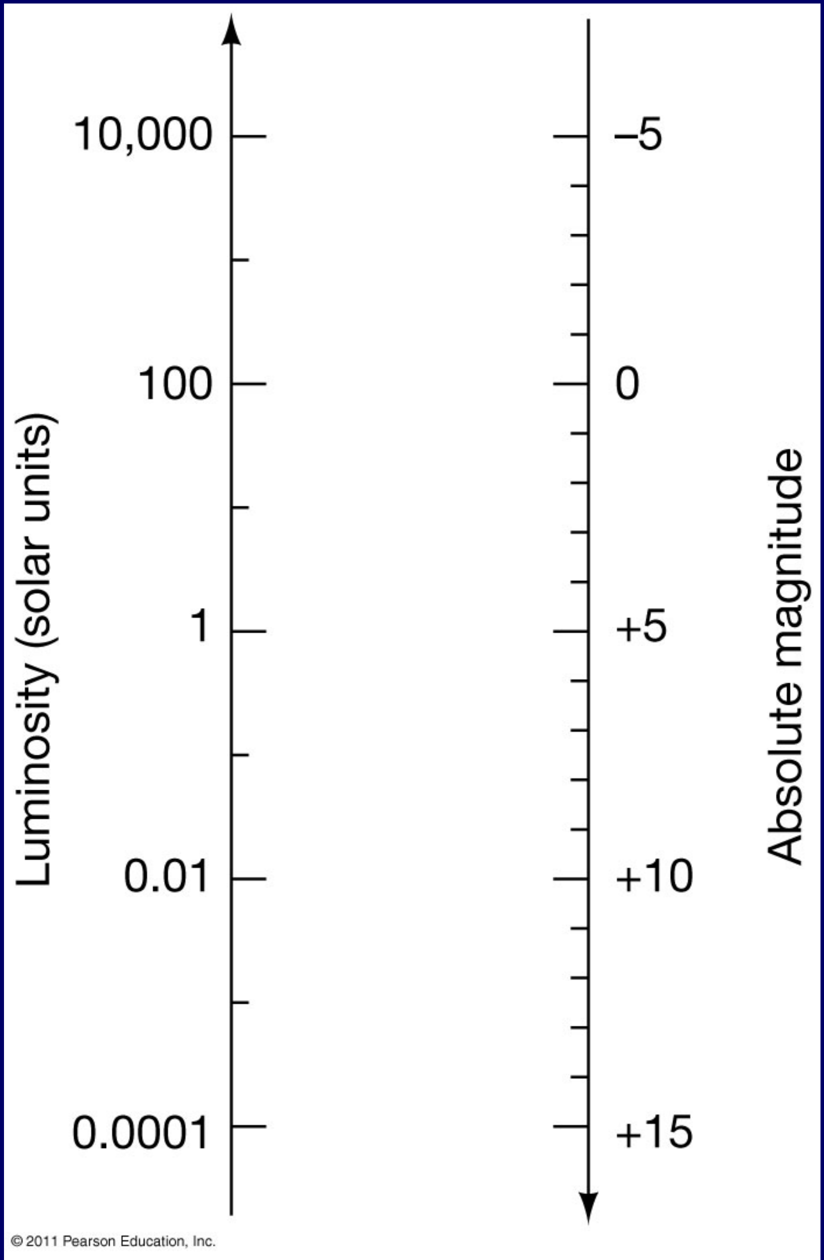
$$M_V = m_V + 5 - 5\ \log_{10}(d\ [\mathrm{pc}])$$

The relation between M_V and L is

$$L = \log_{10}^{-1}\,[(K - M_V)/2.512]$$

Where K is a constant that depends on observation conditions.

Luminosity *vs.* Absolute Magnitude



Converting from magnitude to luminosity in solar units: This graph allows us to perform this conversion simply by reading horizontally. Note that a reduction of 5 in magnitude corresponds to an increase in a factor of 100 in luminosity.

Color and Temperature

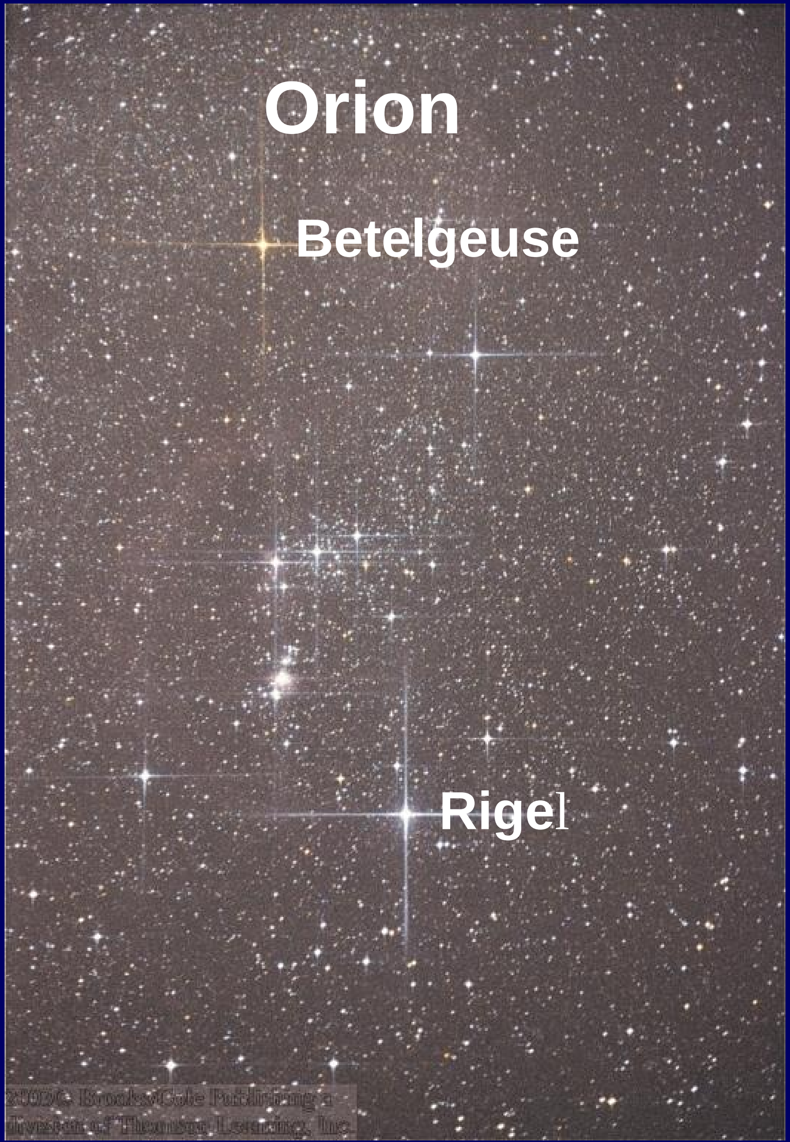
Stars appear in different colors,

from blue (like Rigel)

via green / yellow (like our Sun)

to red (like Betelgeuse).

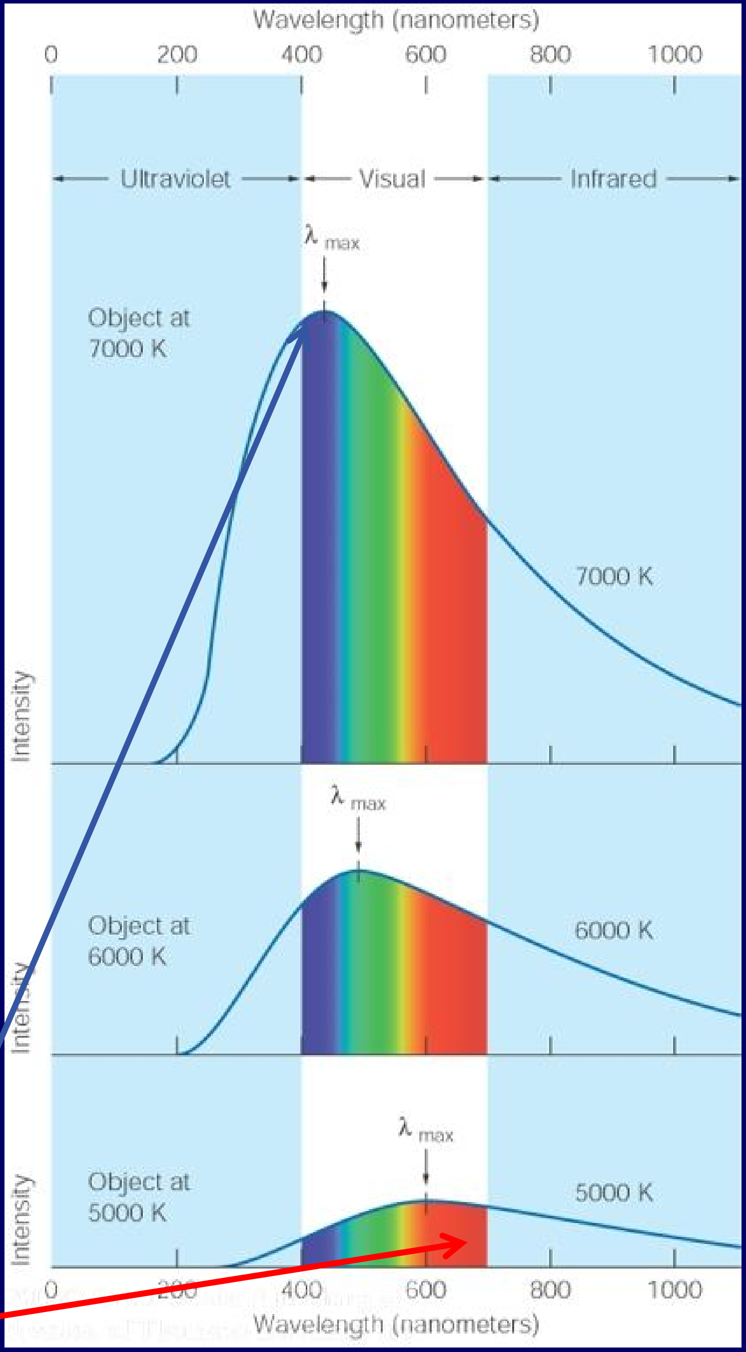
These colors tell us about the star's temperature.



Laws of Thermal Radiation

1. The hotter an object is, the more luminous it is. **Stefan's Law**:
   $$F = \sigma T_K^{\,4}$$
   (where F is power/unit area, σ = 5.67 x 10^{-8} W/m$^2\cdot$K^4 , T_K is the temperature in Kelvin).

2. The peak of the black body spectrum shifts towards shorter wavelengths when the temperature increases. **Wien's displacement law**:
   $$\lambda_{max} \approx 2{,}900{,}000 \text{ nm } / T_K$$

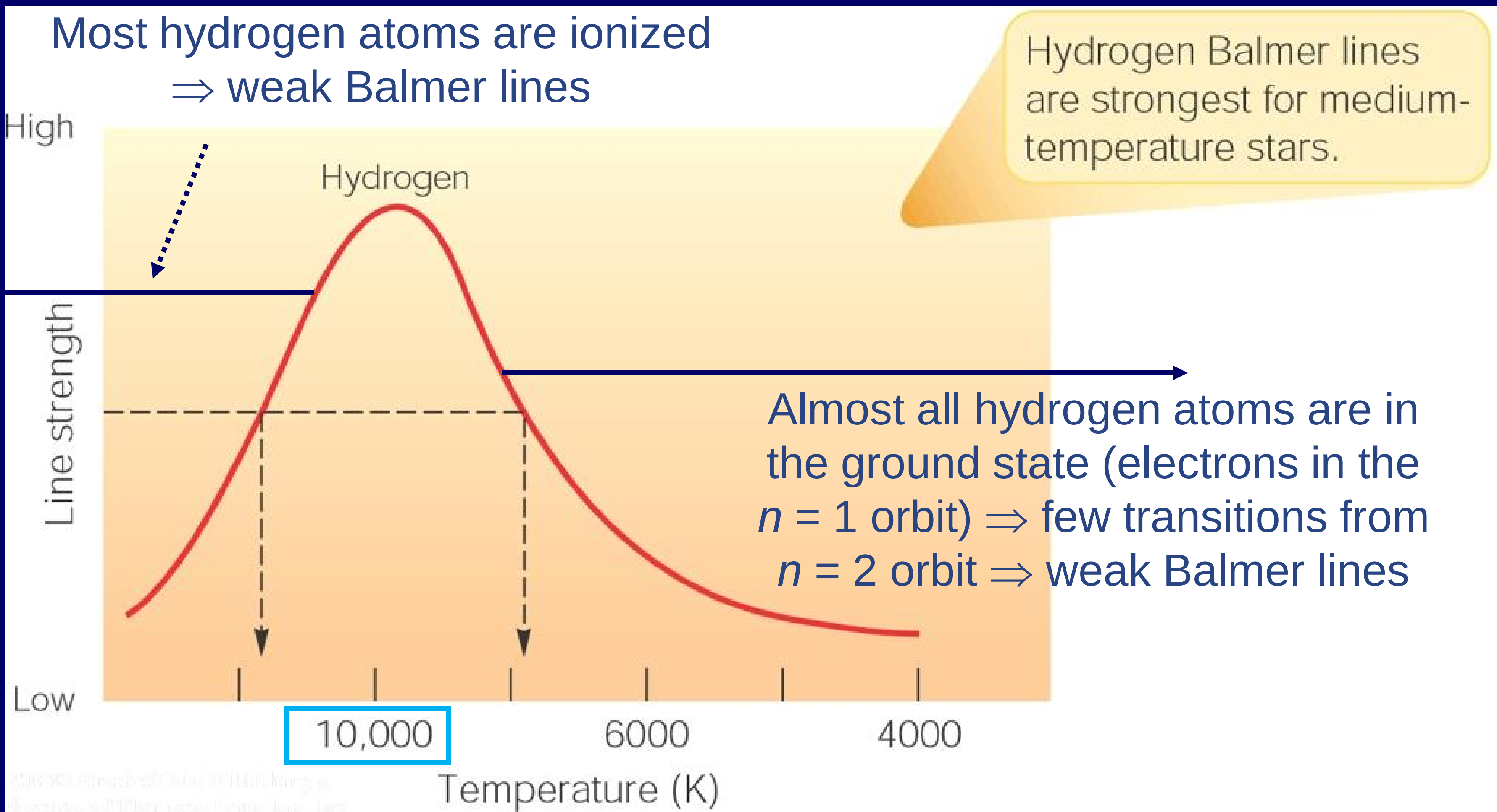


Note how the hottest star looks blue and the coolest star looks red

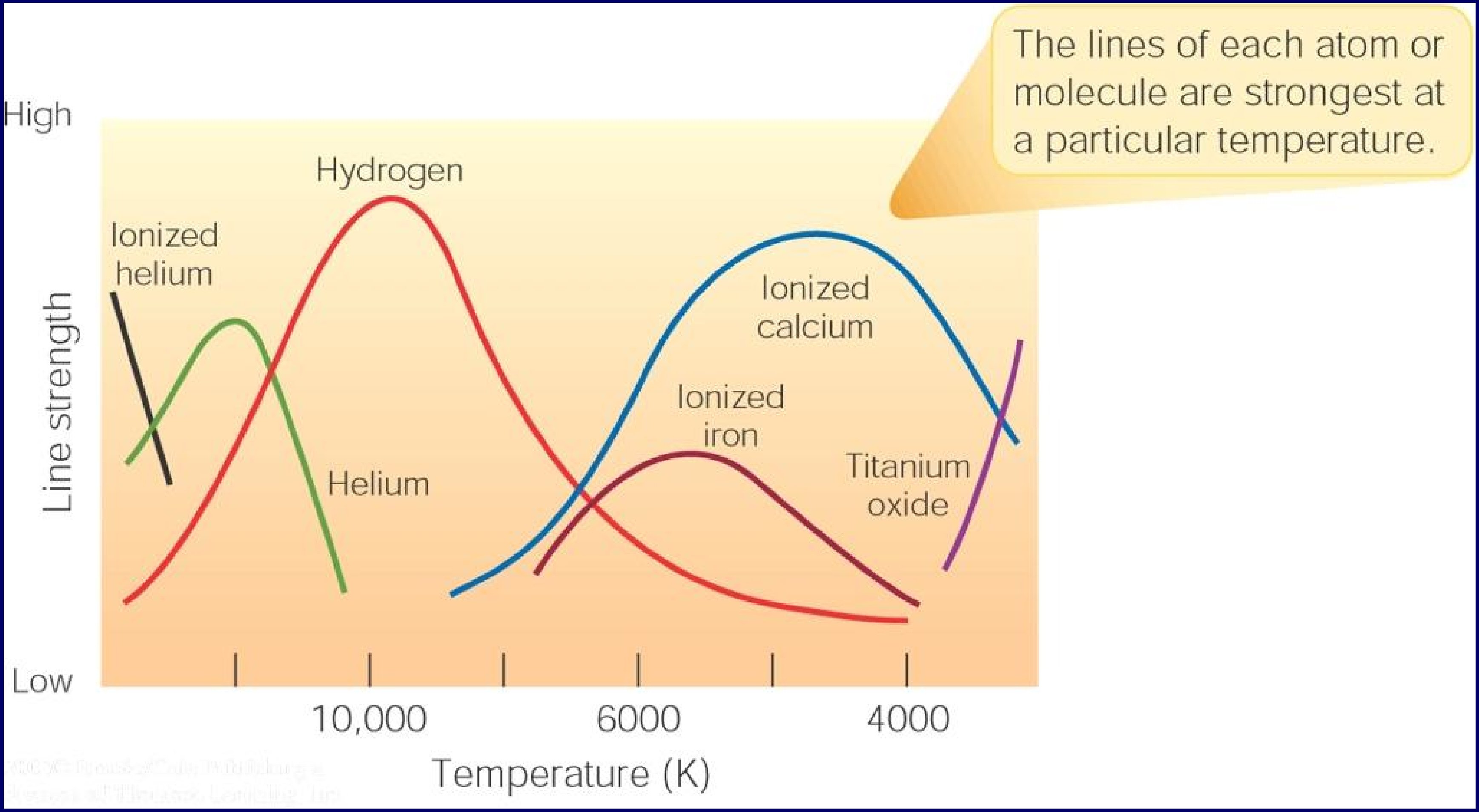
Line Strength and Stellar Type

The Balmer Thermometer

Balmer line strength is sensitive to temperature peaking at ~10^4 K.



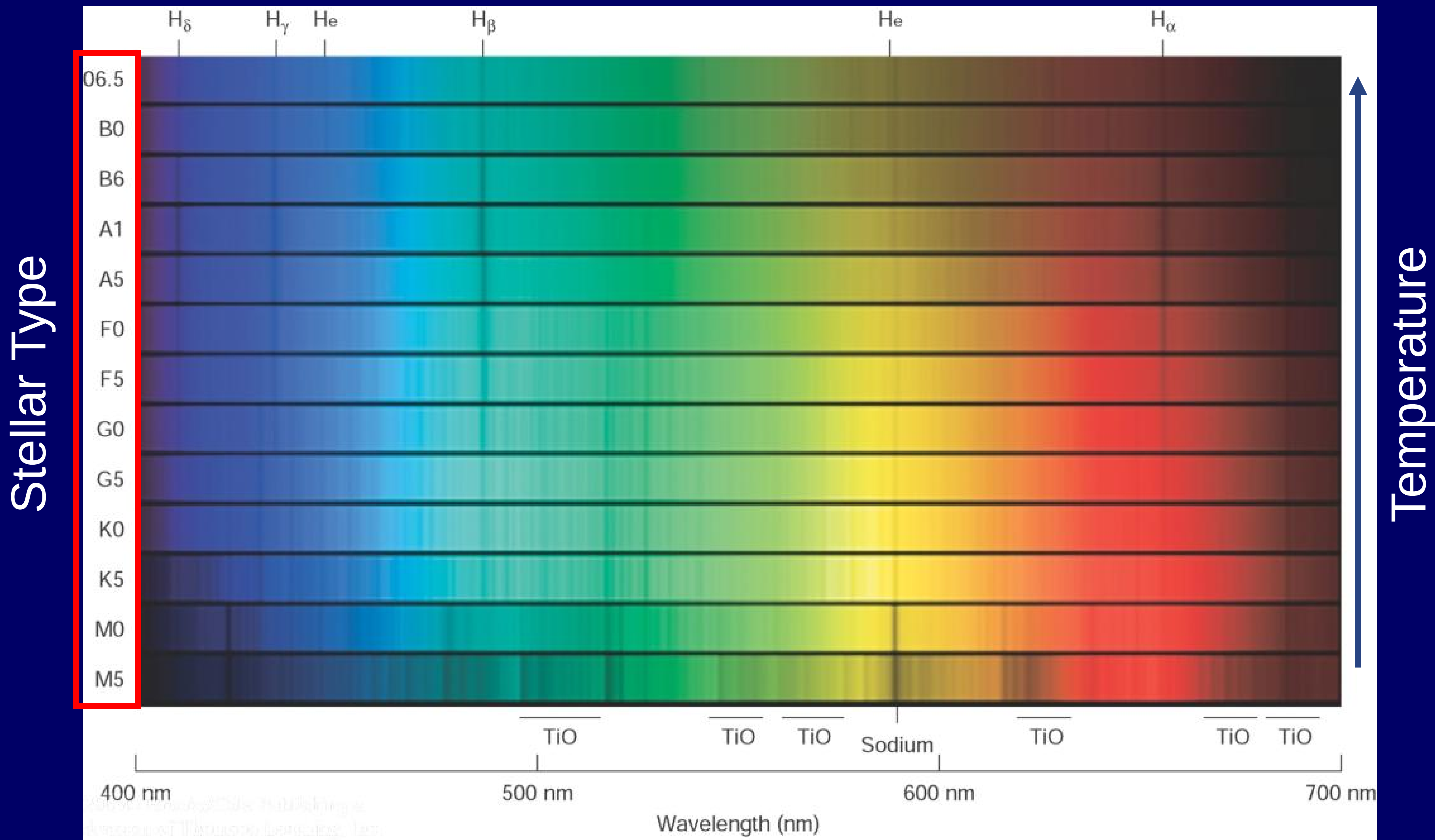
Measuring the Temperatures of Stars Using Line Strength



Different elements, ions and molecules have line strength peaks at different temperatures. Comparing line strengths, we can measure a star's surface temperature!

Spectral Classification of Stars

Different stars show different characteristic sets of absorption lines. These are used to define stellar types.



Spectral Classification of Stars

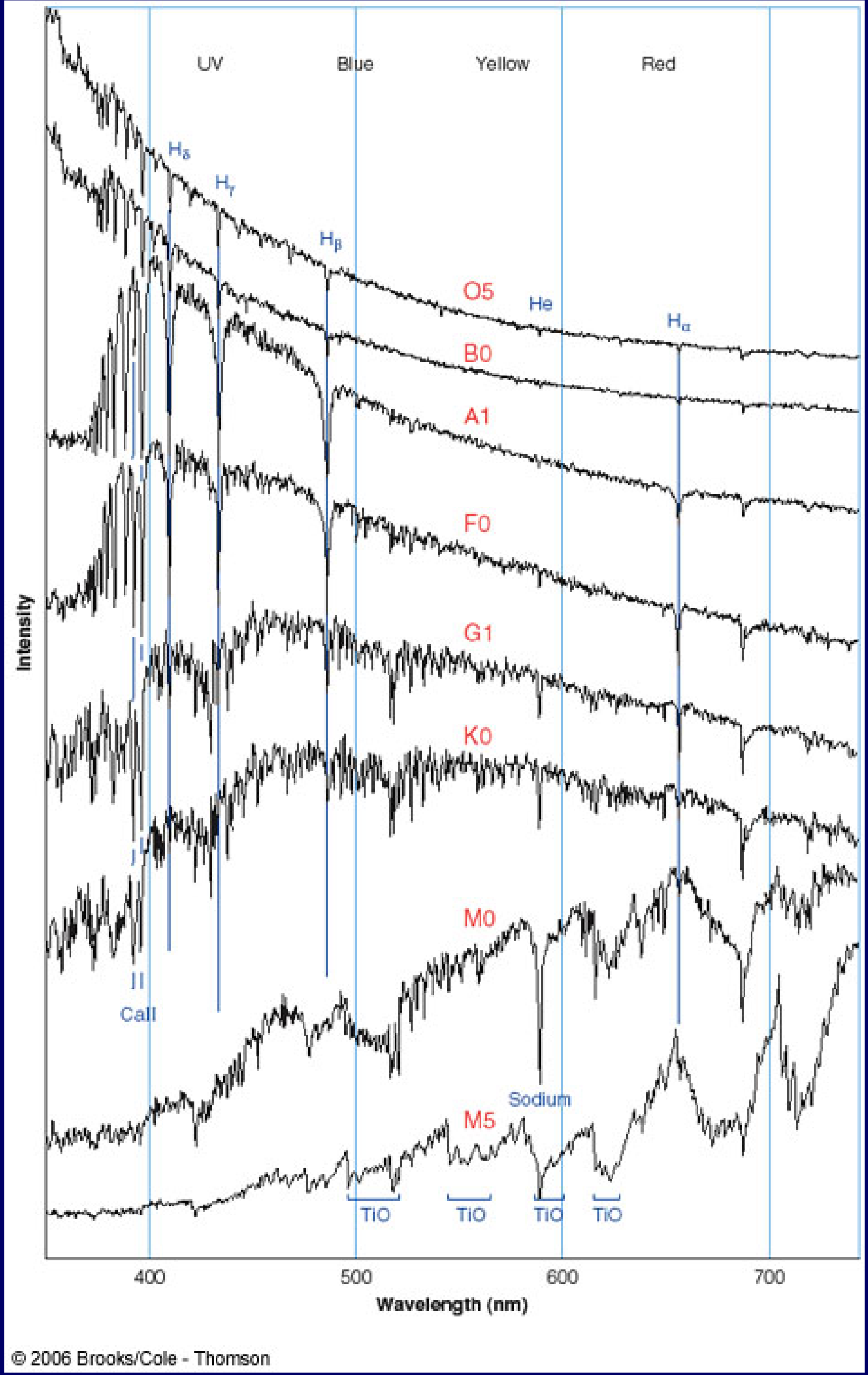
TABLE 17.2 Stellar Spectral Classes

Spectral Class	Approximate Surface Temperature (K)	Noteworthy Absorption Lines	Familiar Examples
O	30,000	Ionized helium strong; multiply ionized heavy elements; hydrogen faint	Mintaka (O9)
B	20,000	Neutral helium moderate; singly ionized heavy elements; hydrogen moderate	Rigel (B8)
A	10,000	Neutral helium very faint; singly ionized heavy elements; hydrogen strong	Vega (A0), Sirius (A1)
F	7000	Singly ionized heavy elements; neutral metals; hydrogen moderate	Canopus (F0)
G	6000	Singly ionized heavy elements; neutral metals; hydrogen relatively faint	Sun (G2), Alpha Centauri (G2)
K	4000	Singly ionized heavy elements; neutral metals strong; hydrogen faint	Arcturus (K2), Aldebaran (K5)
M	3000	Neutral atoms strong; molecules moderate; hydrogen very faint	Betelgeuse (M2), Barnard's star (M5)

Mnemonics to remember the spectral sequence

O	B	A	F	G	K	M
Oh	Be	A	Fine	Girl/Guy	Kiss	Me
Oh	Boy,	An	F	Grade	Kills	Me
Only	Bad	Astronomers	Forget	Generally	Known	Mnemonics

Stellar Spectra



O
B
A
F
G
K
M

Surface temperature

Note that these electronically measured spectra show the Planck continuum spectrum with the absorption spectra as dips from the continuum

The Composition of Stars

From the relative strength of absorption lines (carefully accounting for their temperature dependence), one can infer the surface composition of stars.

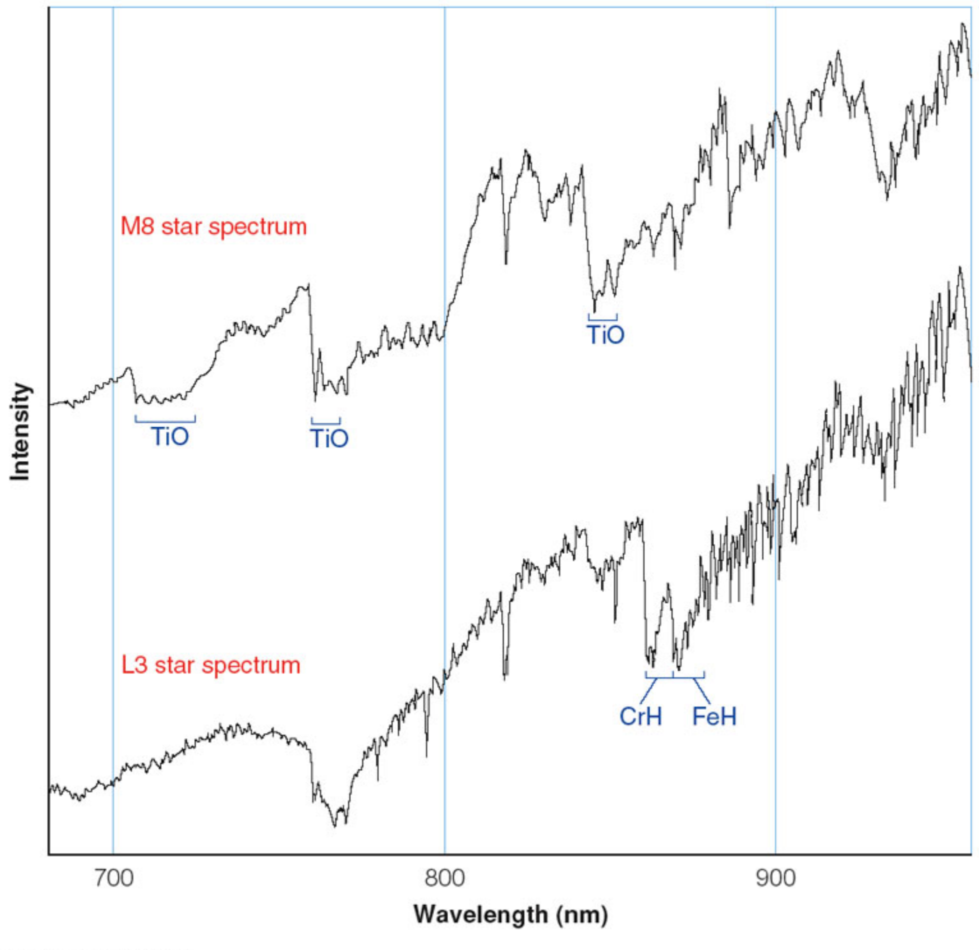


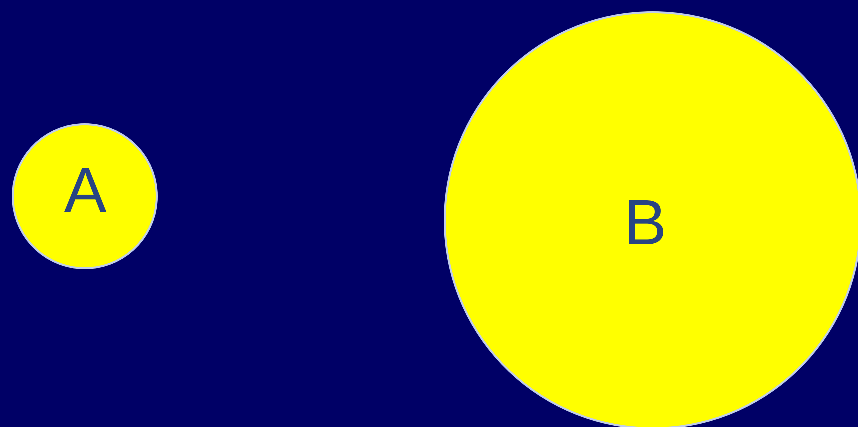
Table 6-2 | The Most Abundant Elements in the Sun

Element	Percentage by Number of Atoms	Percentage by Mass
Hydrogen	91.0	70.9
Helium	8.9	27.4
Carbon	0.03	0.3
Nitrogen	0.008	0.1
Oxygen	0.07	0.8
Neon	0.01	0.2
Magnesium	0.003	0.06
Silicon	0.003	0.07
Sulfur	0.002	0.04
Iron	0.003	0.1

The Radius of a Star

We already know: luminosity increases with surface temperature ($\sim T^4$) (see the section on Light); hotter stars are brighter.

But luminosity also increases with size:



Star B will be brighter than star A at the same temperature.

$\Rightarrow$ **Luminosity is proportional to the radius squared, $L \propto R^2$.**

Quantitatively: $$\boxed{L = \underbrace{4\pi R^2}_{\text{Surface area of the star}}\ \underbrace{\sigma T^4}_{\text{Energy flux of a blackbody}}}$$

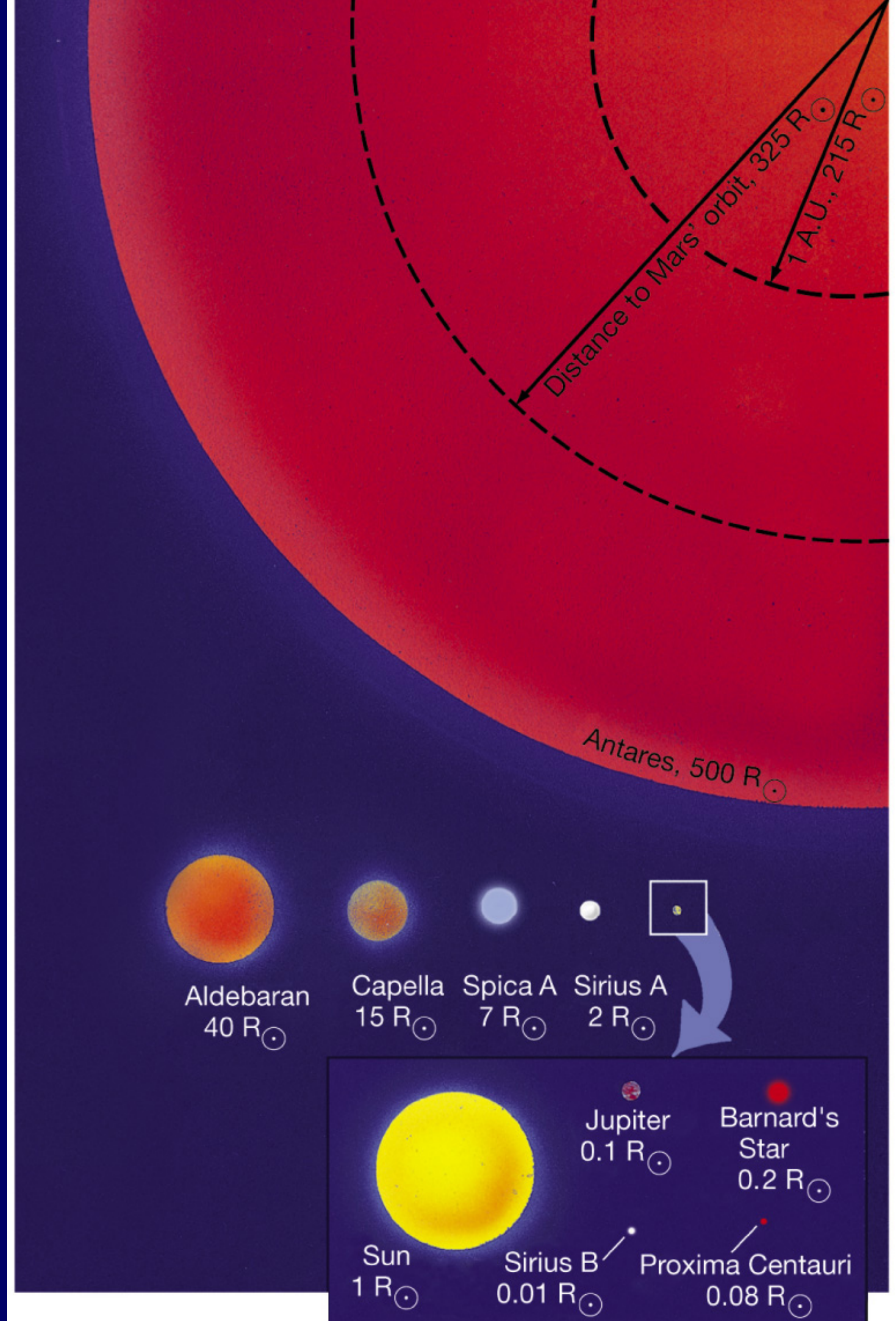
Example:

Polaris is just about the same spectral type (G) (and thus surface temperature) as our Sun, but it is 10,000 times brighter than our Sun. This must be caused by Polaris having a larger radius than the Sun. Polaris must be 100 times larger than the Sun because $R^2 = 100^2 = 10,000$ produces a luminosity 10,000 times brighter than our Sun.

Stellar Radii

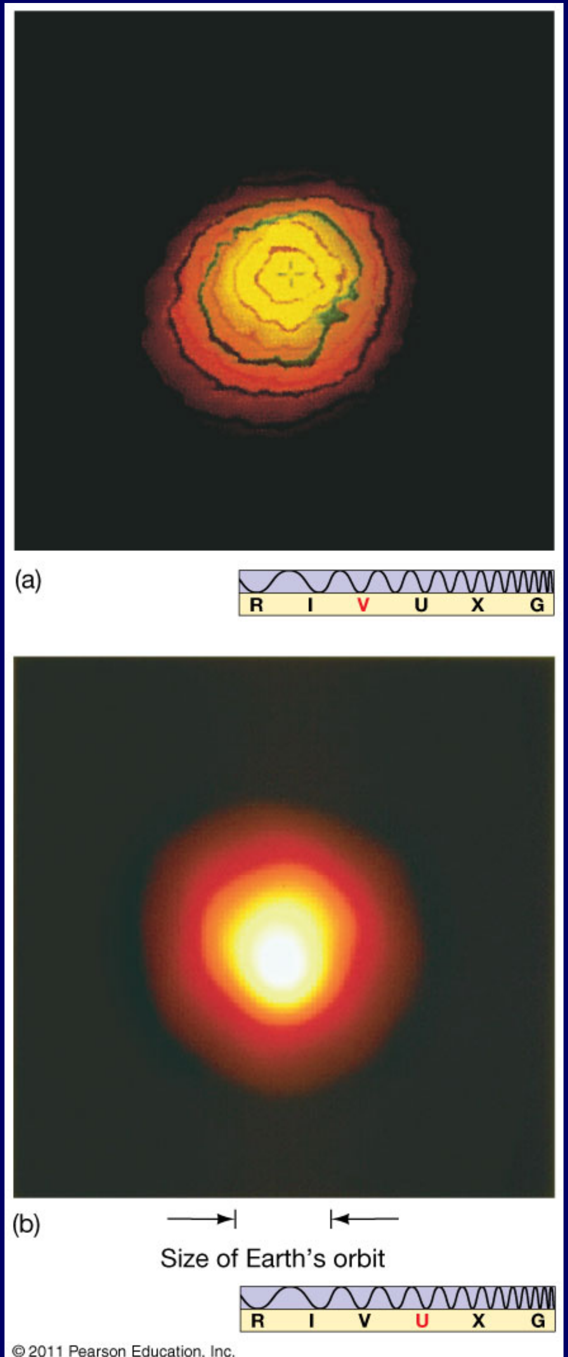
Stellar radii vary widely:

- **Dwarf** stars have radii equal to, or less than, the Sun's radius.
- **Giant** stars have radii between 10 and 100 times the Sun's radius.
- **Supergiant** stars have radii more than 100 times the Sun's radius.
- Note: We generally use the Sun's radius $R_\odot$ as our standard distance



Stellar Sizes

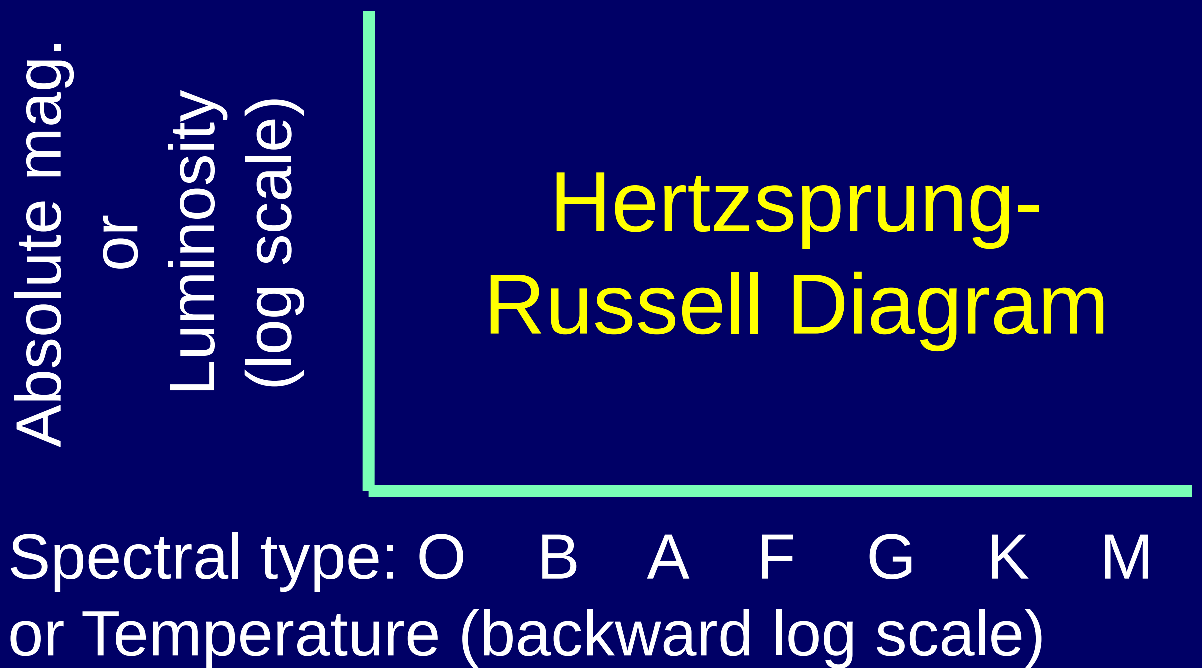
A few very large, very close stars can be imaged directly using speckle interferometry; this is Betelgeuse. The first speckle interferometry image of Betelgeuse consisted of about 15 pixels. a) Very good resolution. b) Even better resolution



Organizing the Family of Stars: The Hertzsprung-Russell Diagram

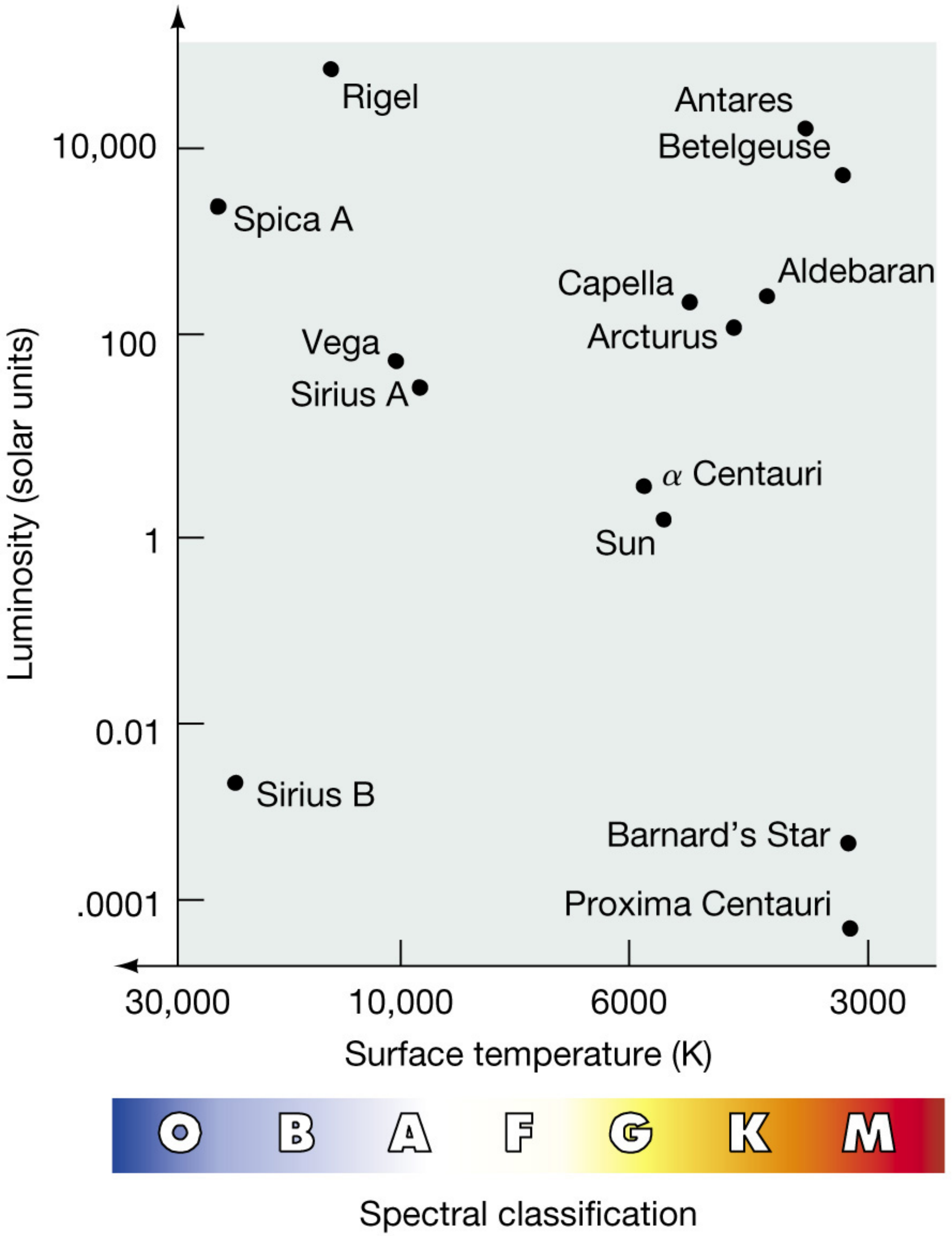
We know that stars have different temperatures, different luminosities, and different sizes.

To bring some order to this zoo of different stars Ejnar Hertzsprung and Henry Norris Russell organized them in a diagram of Luminosity (Absolute magnitude) versus Temperature (Spectral type)



The Hertzsprung–Russell Diagram

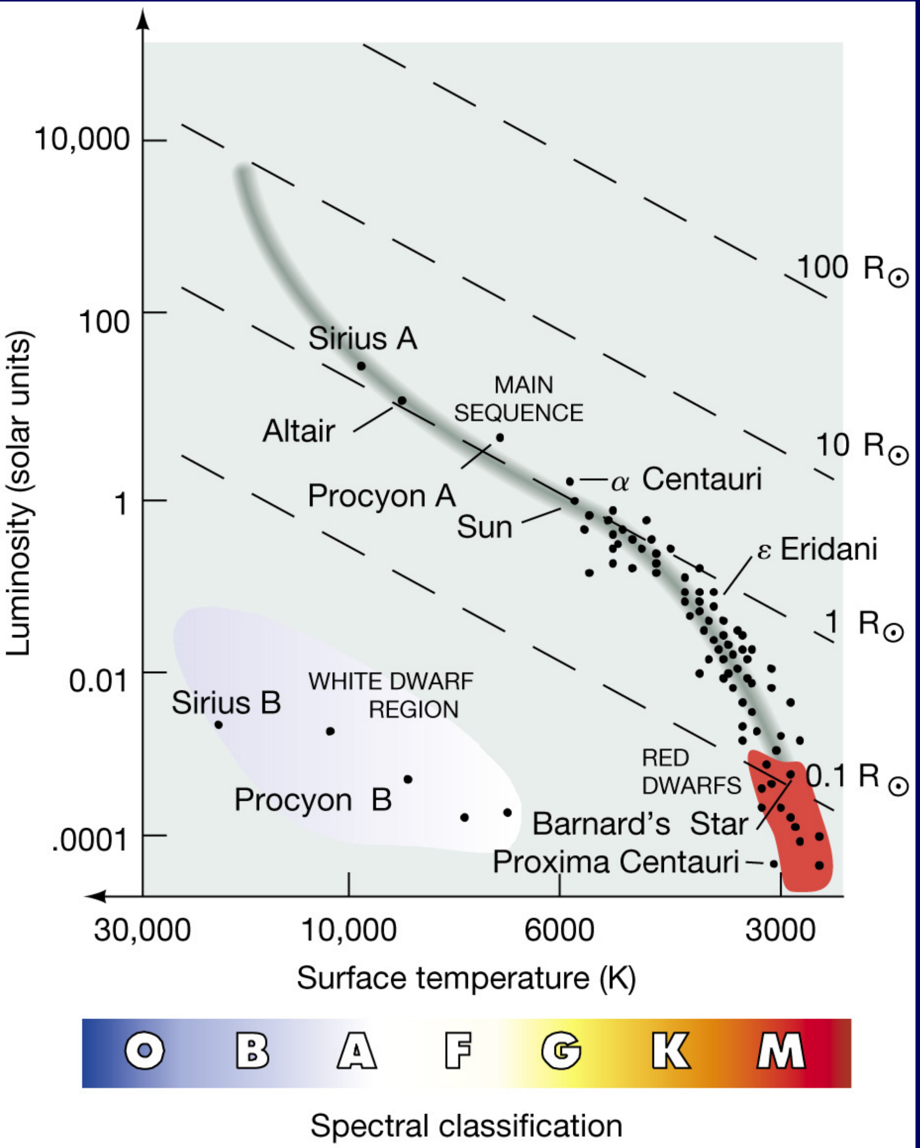
This is an H-R diagram of a few prominent stars (high apparent brightness). With this small sample we would conclude that the H-R diagram is not helpful in showing any stellar organization.



The Hertzsprung–Russell Diagram

Once many stars are plotted on an H–R diagram, a pattern begins to form:

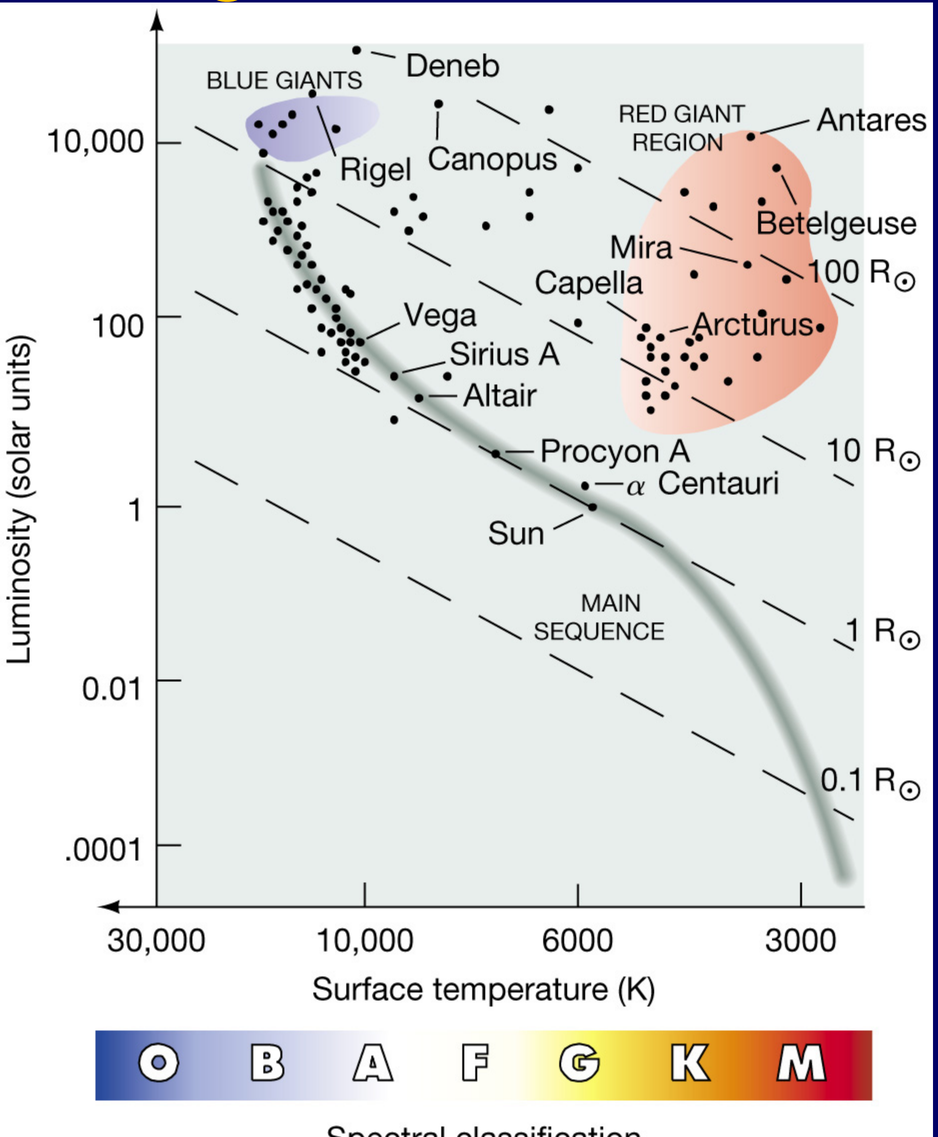
- **These are the 80 closest stars to us; note the dashed lines of constant radius.**
- **The darkened curve is called the main sequence because this is where most stars are.**
- **The white dwarf region is also indicated; these stars are hot but not very luminous because they are quite small.**



The Hertzsprung–Russell Diagram

An H–R diagram of the 100 brightest stars looks quite different:

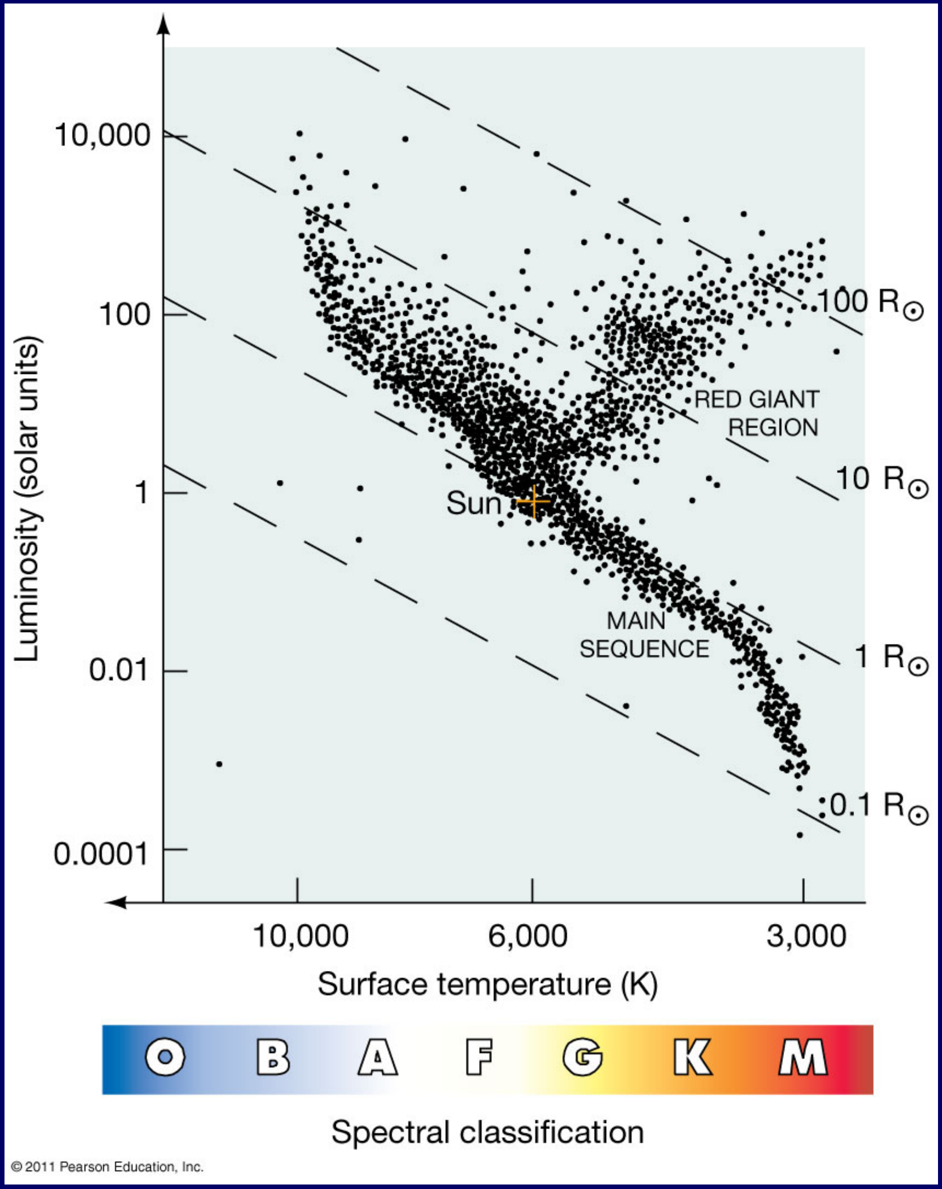
- These stars are all more luminous than the Sun.
- Two new categories appear here – the red giants and the blue giants.
- Clearly, the brightest stars in the sky appear bright because of their enormous luminosities, not their proximity.



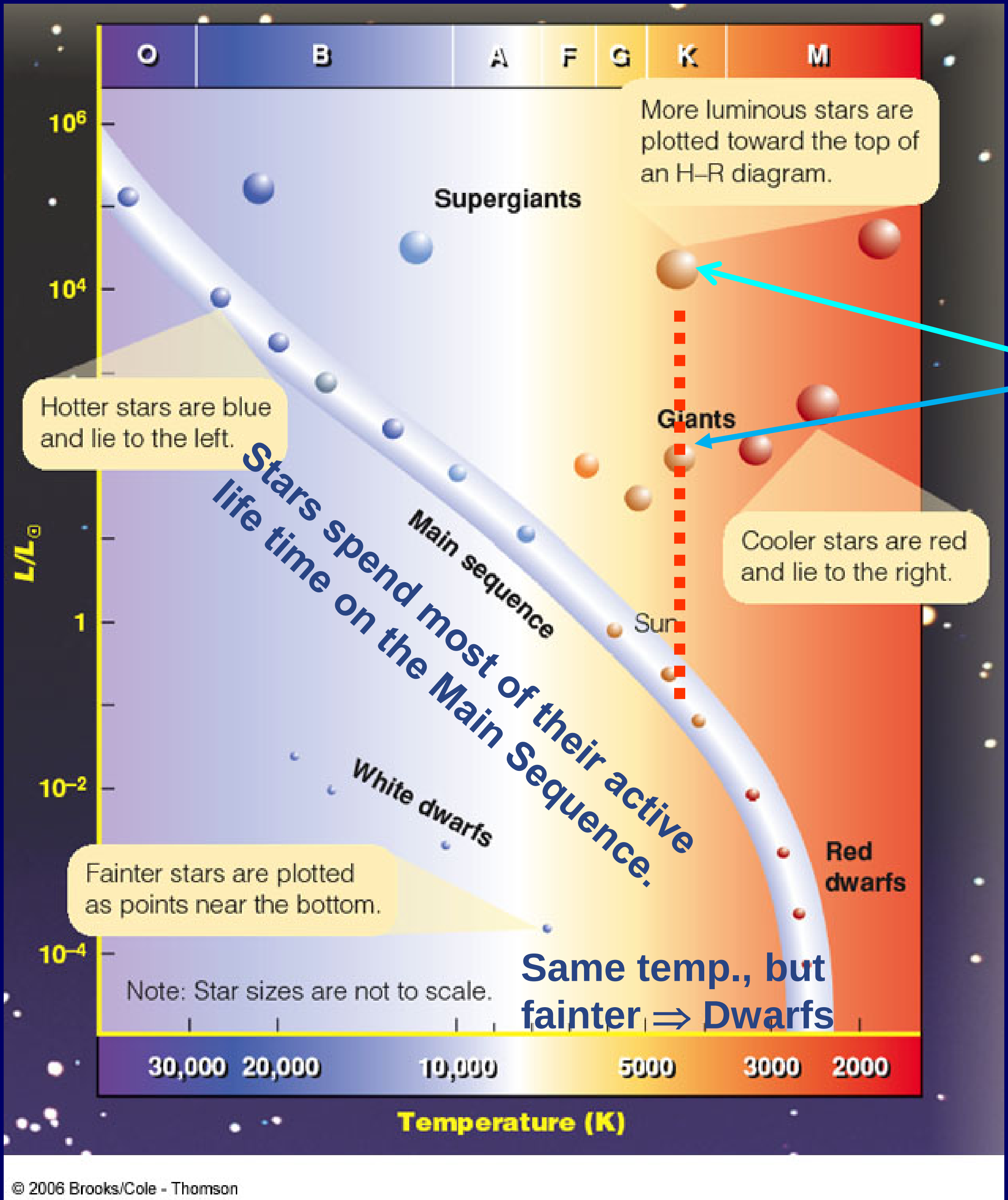
The Hertzsprung-Russell Diagram

This is an H–R diagram of about 20,000 stars. The main sequence is clear, as well as the red giant region.

About 90% of stars lie on the main sequence; 9% are red giants and 1% are white dwarfs.



The Hertzsprung-Russell Diagram



Same temperature, but much brighter than MS stars ⇒ Must be much larger

⇒ Giant Stars

Extending the Cosmic Distance Scale: Spectroscopic Parallax

Spectroscopic parallax has nothing to do with parallax, but it does use spectroscopy to find the distance to a main-sequence star.

Method:

1. Measure the star's apparent magnitude b and spectral type (OBFGKM)
2. On the H-R diagram, use the spectral type to find the luminosity L
3. Apply the inverse-square law to find distance

$$b = L/4\pi d^2$$

$$d = (L/4\pi b)^{1/2}$$

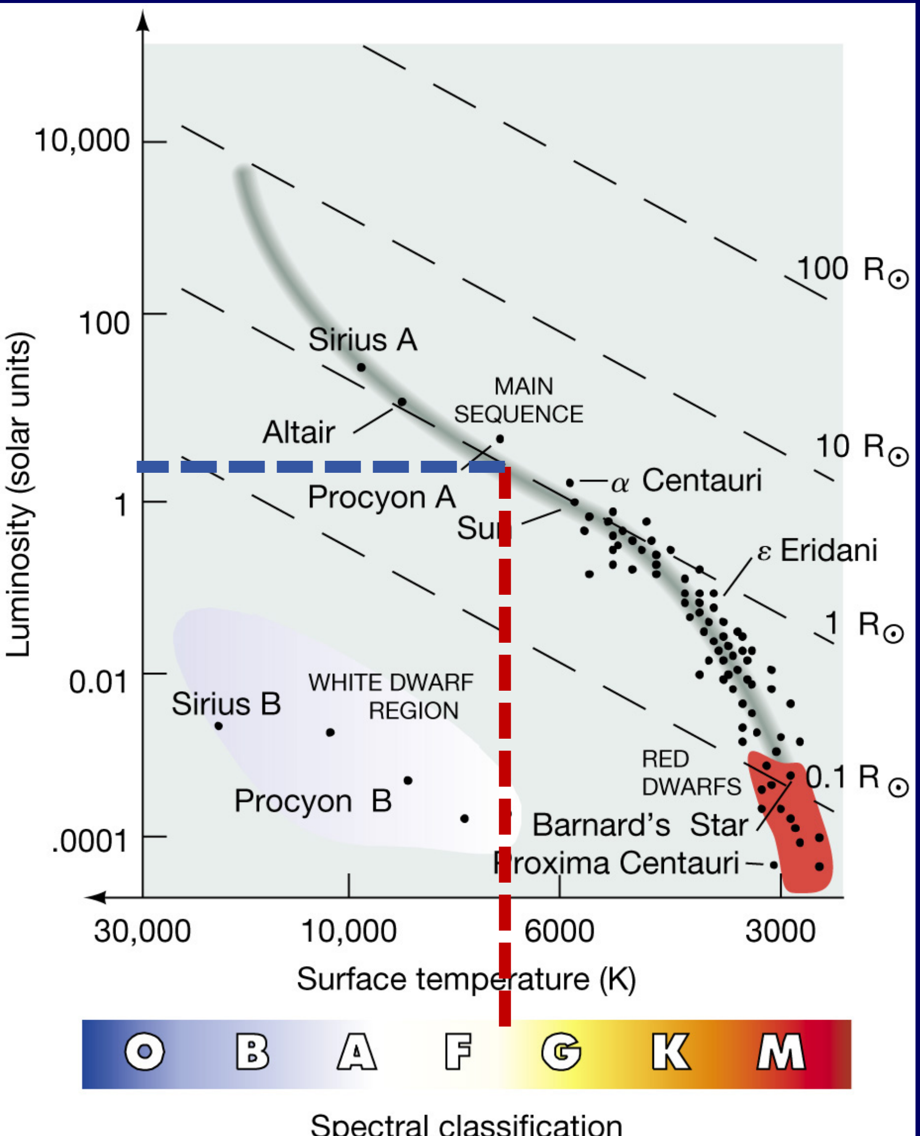
Spectroscopic Parallax

Example

Stellar type: F9

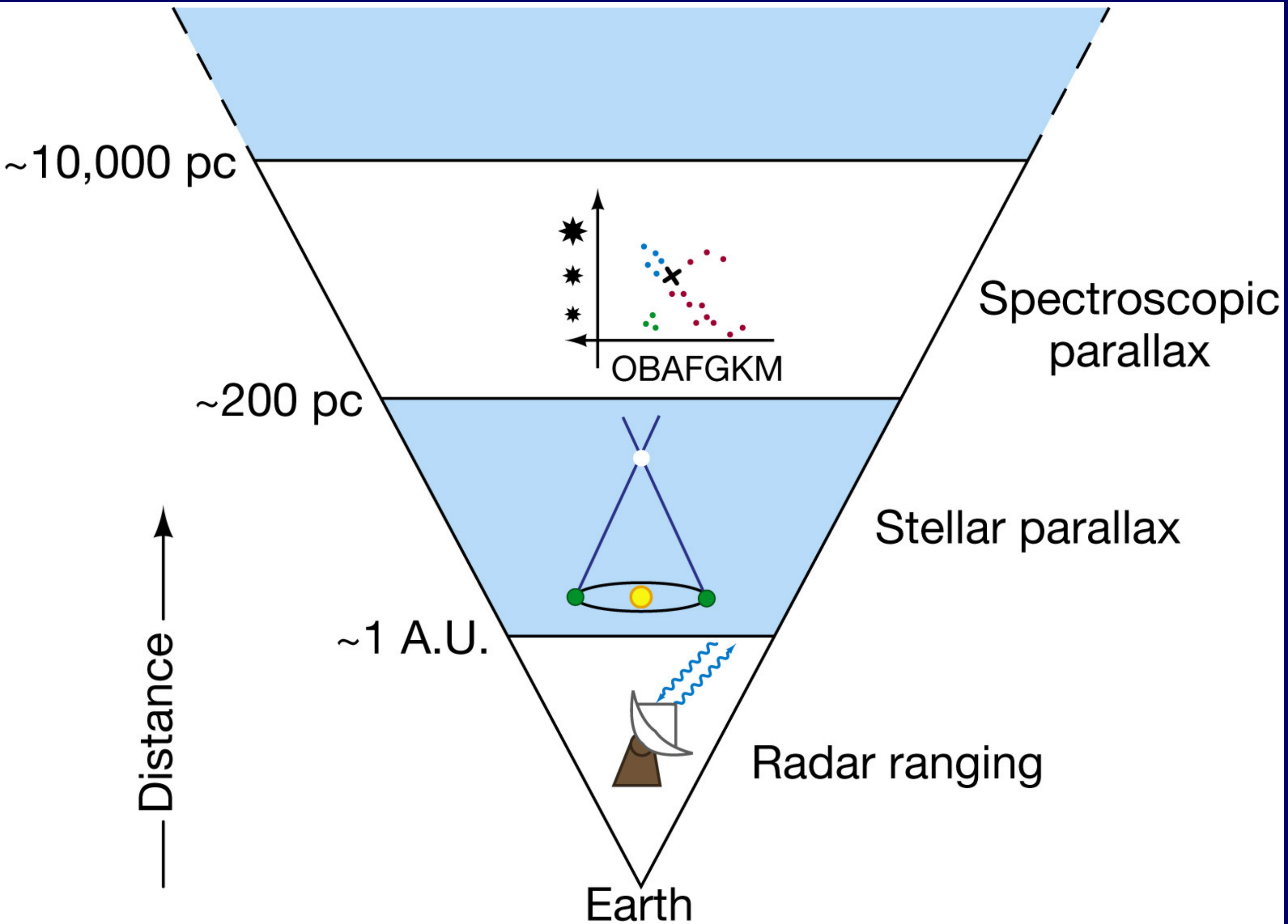
Luminosity: 1.8 $L_\odot$

Distance: $d = (1.8\ L_\odot/4\pi b)^{1/2}$



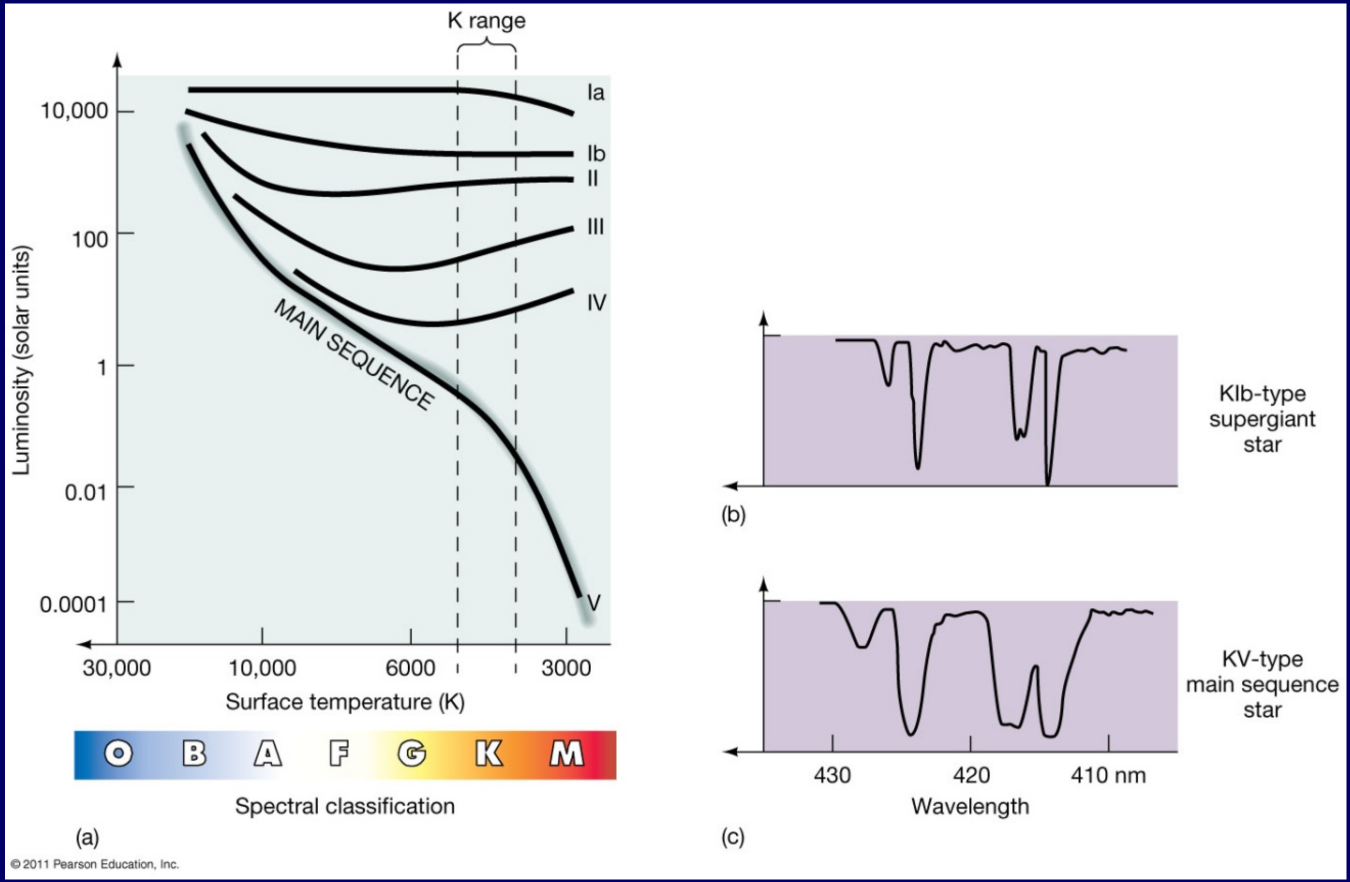
Extending the Cosmic Distance Scale

Spectroscopic parallax can extend the cosmic distance scale to several thousand parsecs

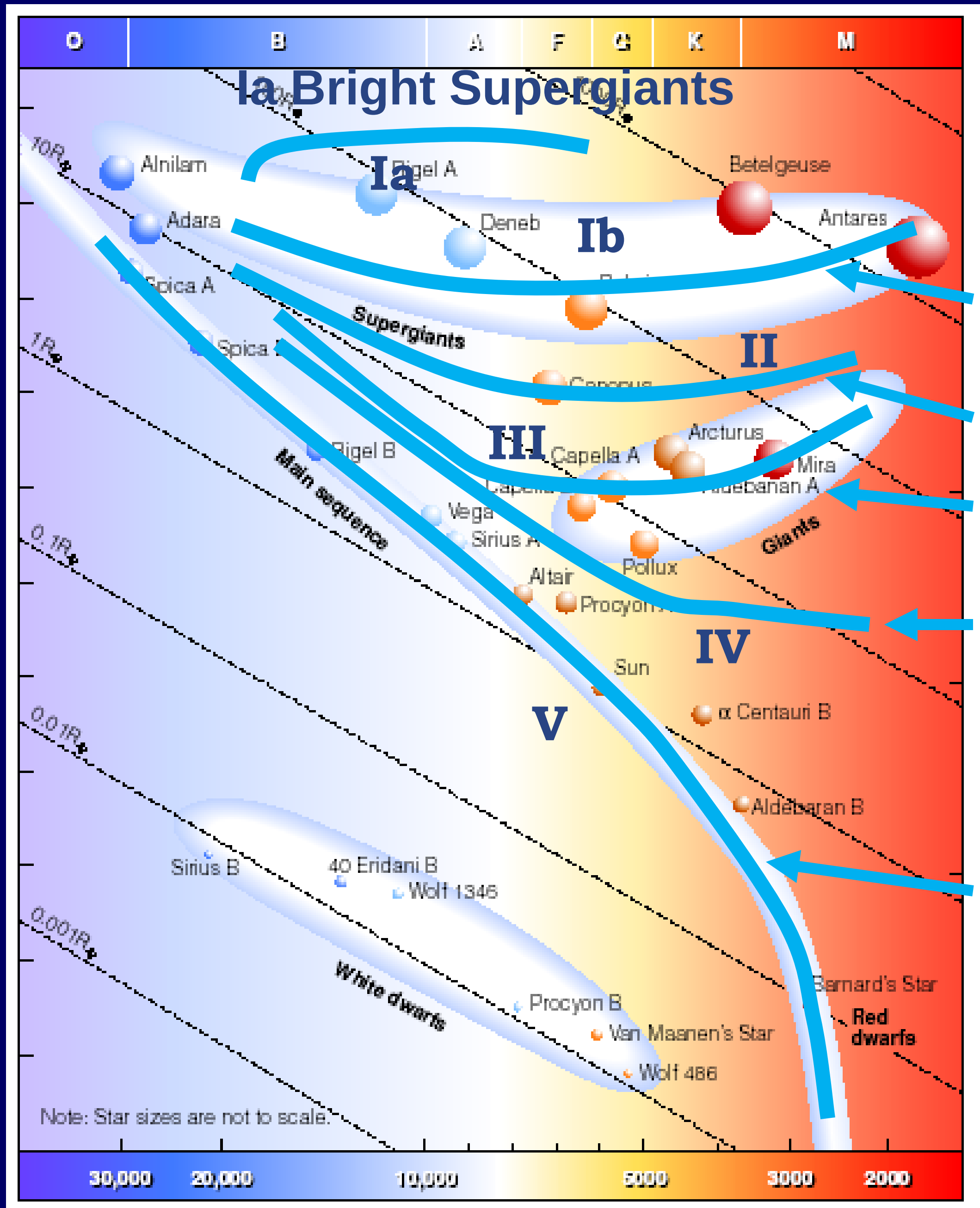


Extending the Cosmic Distance Scale

The spectroscopic parallax calculation can be misleading if the star is not on the main sequence. The width of spectral lines can be used to define luminosity classes.



In this way, giants and supergiants can be distinguished from main-sequence stars



Luminosity Classes

Ib Supergiants

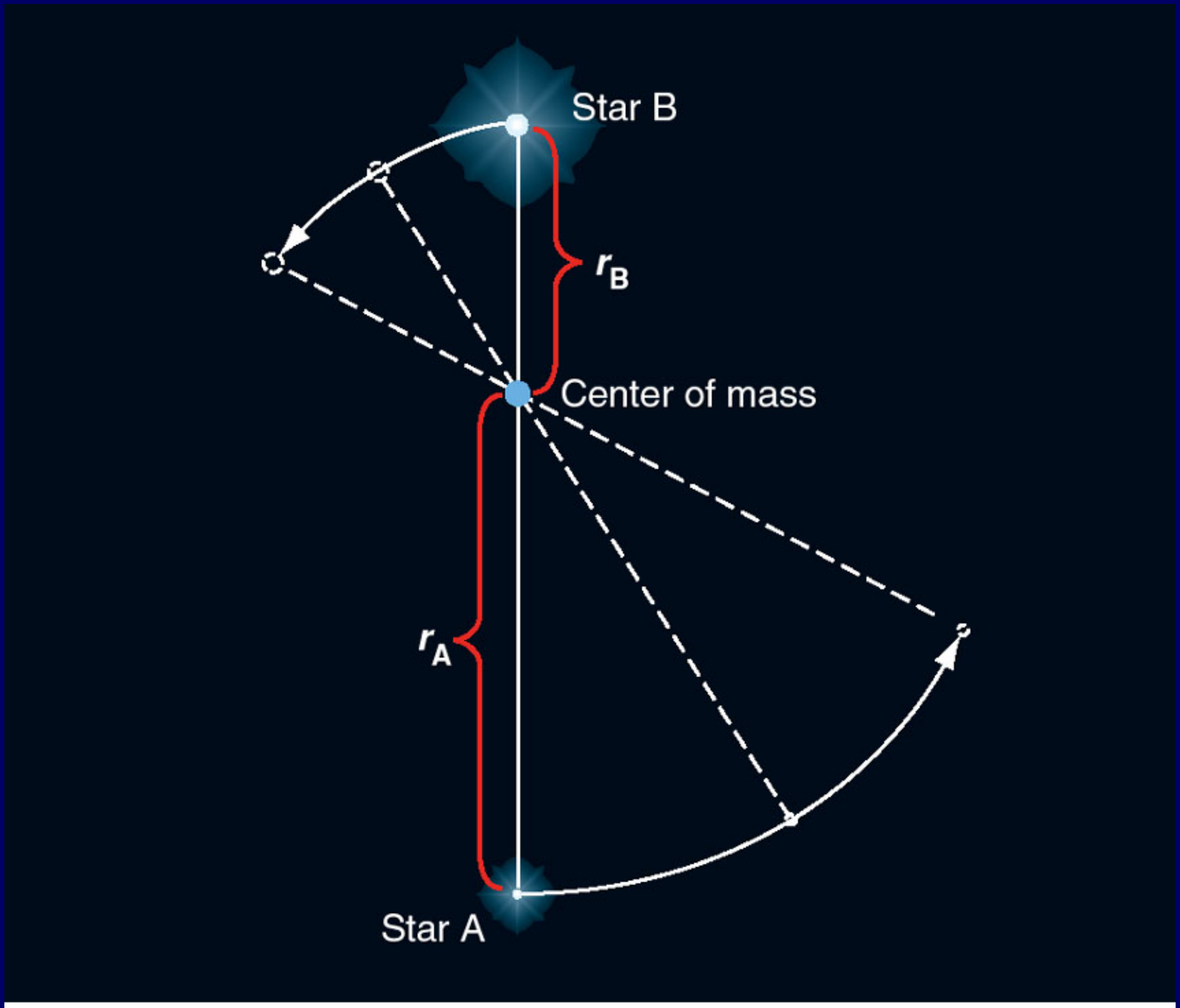
II Bright Giants

III Giants

IV Subgiants

V Main-Sequence Stars

Binary Stars

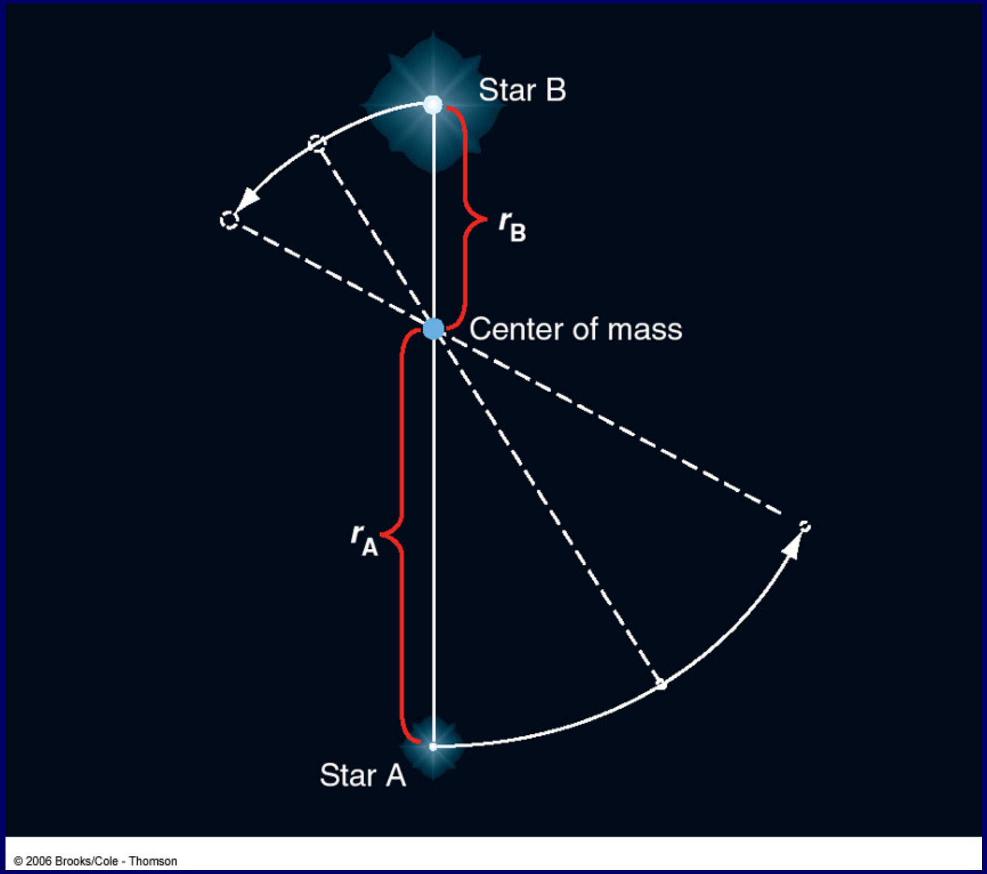


More than 50% of all stars in our Milky Way are not single stars, but belong to binaries:

> **Pairs or systems of multiple stars which orbit their common center of mass.**

If we can measure and understand their orbital motions, we can calculate their stellar masses.

The Center of Mass

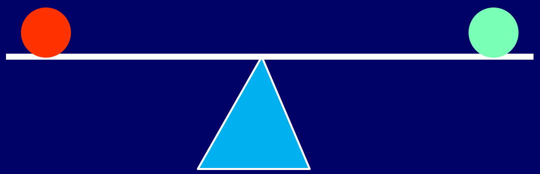


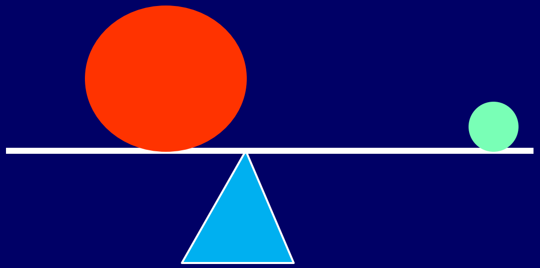
The center of mass is the balance point of the system.

If both masses are equal $\Rightarrow$ center of mass is in the middle,

$$r_A = r_B.$$

The more unequal the masses, the more the c.m. shifts toward the more massive star.





Calculating Stellar Masses

Recall Kepler's 3rd Law:

$$P_y^2 = a_{AU}^3/1\ m_\odot$$

for the solar system which has a star with 1 solar mass in the center.

We find almost the same law for binary stars with masses M_A and M_B

$$\boxed{M_A + M_B = \frac{a_{AU}^3}{P_y^2}}$$

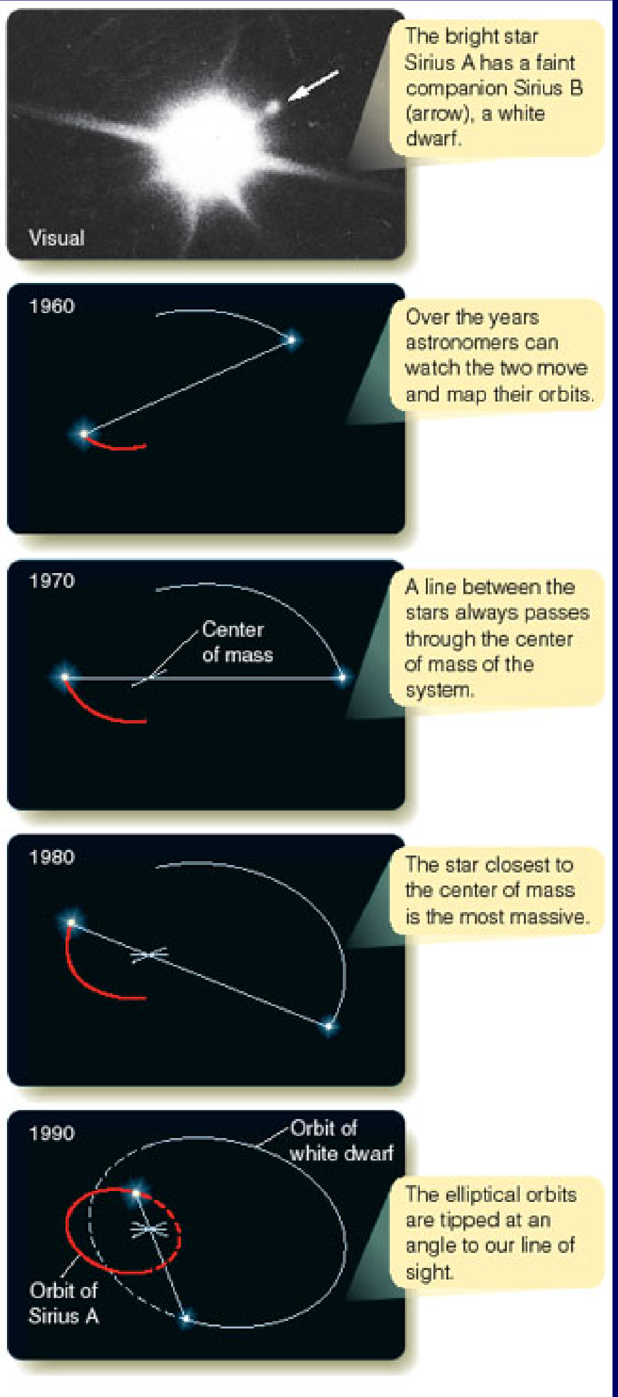
(M_A and M_B in units of solar masses)

Visual Binaries

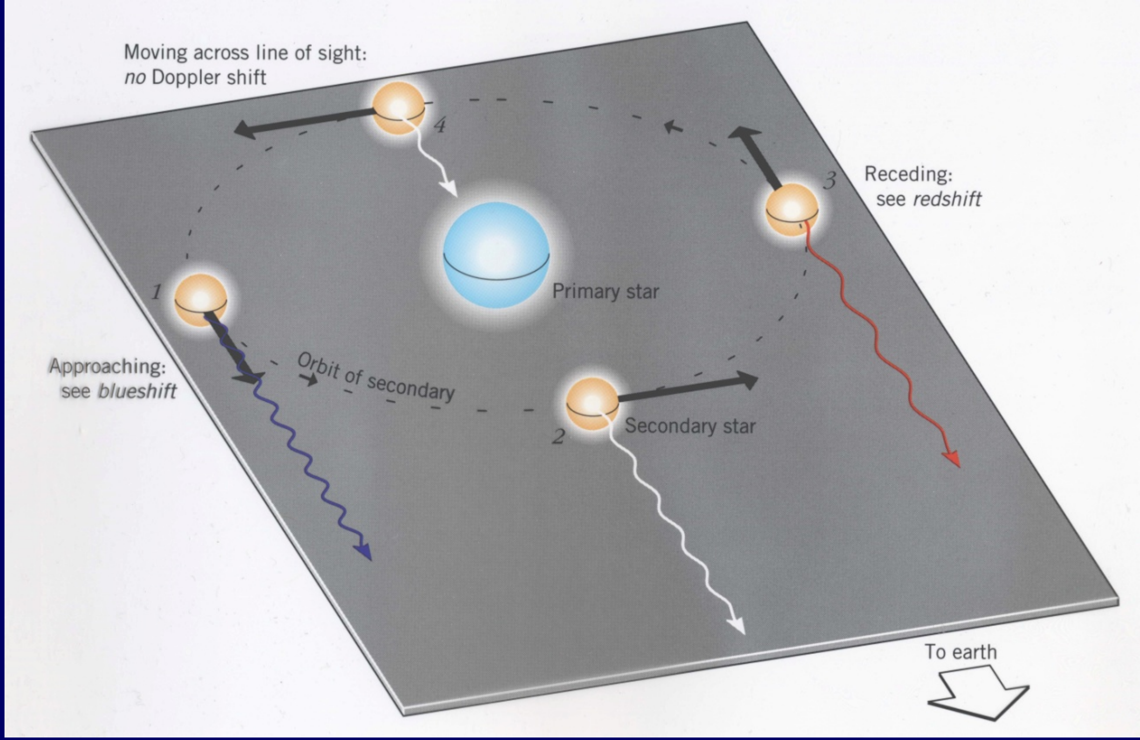
The ideal case:

Both stars can be seen directly, and their separation and relative motion can be followed directly.

Unfortunately, visual binaries usually have long periods and we often have large uncertainties in measuring them.



Spectroscopic Binaries



The binary separation a cannot be measured directly because the stars are too close to each other.

However, in spectroscopic binaries, the stars show Doppler shifts from the radial velocities of the two stars.

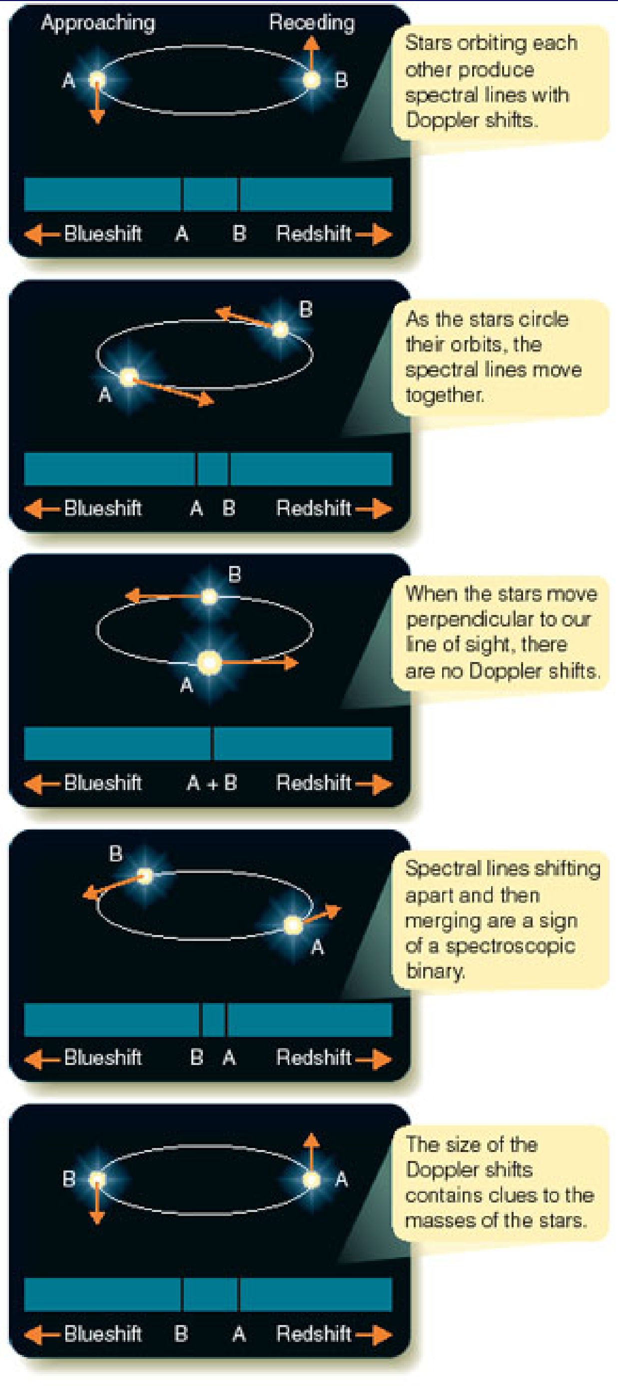
By measuring these Doppler shifts we can determine a limit on the separation and thus the masses can be inferred in the most common cases.

Spectroscopic Binaries

- **The approaching star produces blue-shifted lines; the receding star produces red-shifted lines in the spectrum.**
- **Doppler shift $\Rightarrow$ Measurement of radial velocities v_r. Transverse v_t must be estimated.** $v = (v_r^2 + v_t^2)^{1/2}$
- **Measure the period P by timing the cycle through one or more periods.**
- **Estimate of separation**

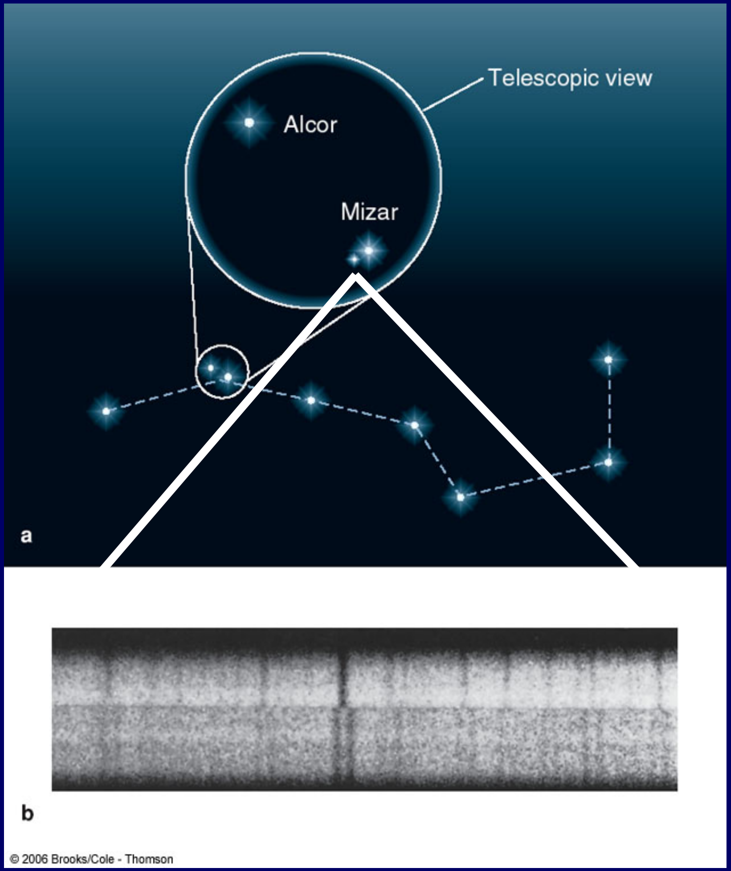
  $$v \approx \pi a/P \quad \Rightarrow \quad a \approx Pv/\pi$$

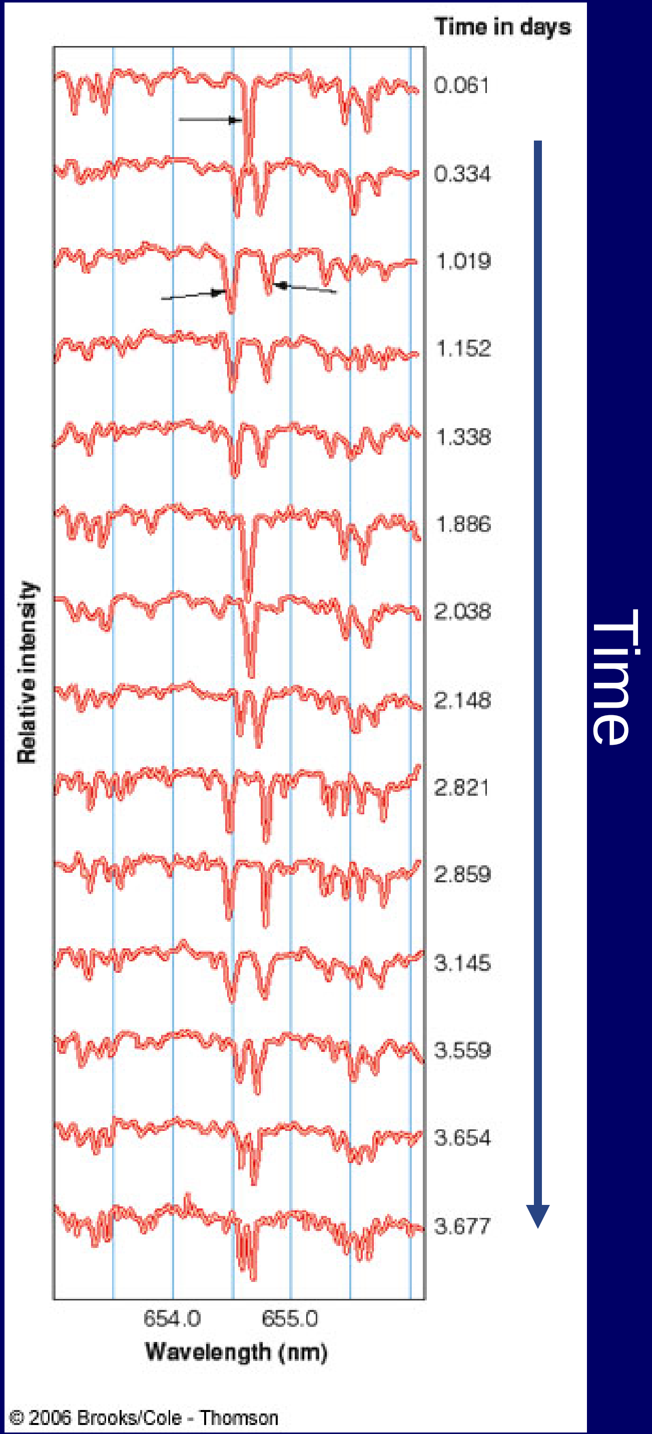
- **Estimate of masses using a and P**



Spectroscopic Binaries

Typical sequence of spectra from a spectroscopic binary system





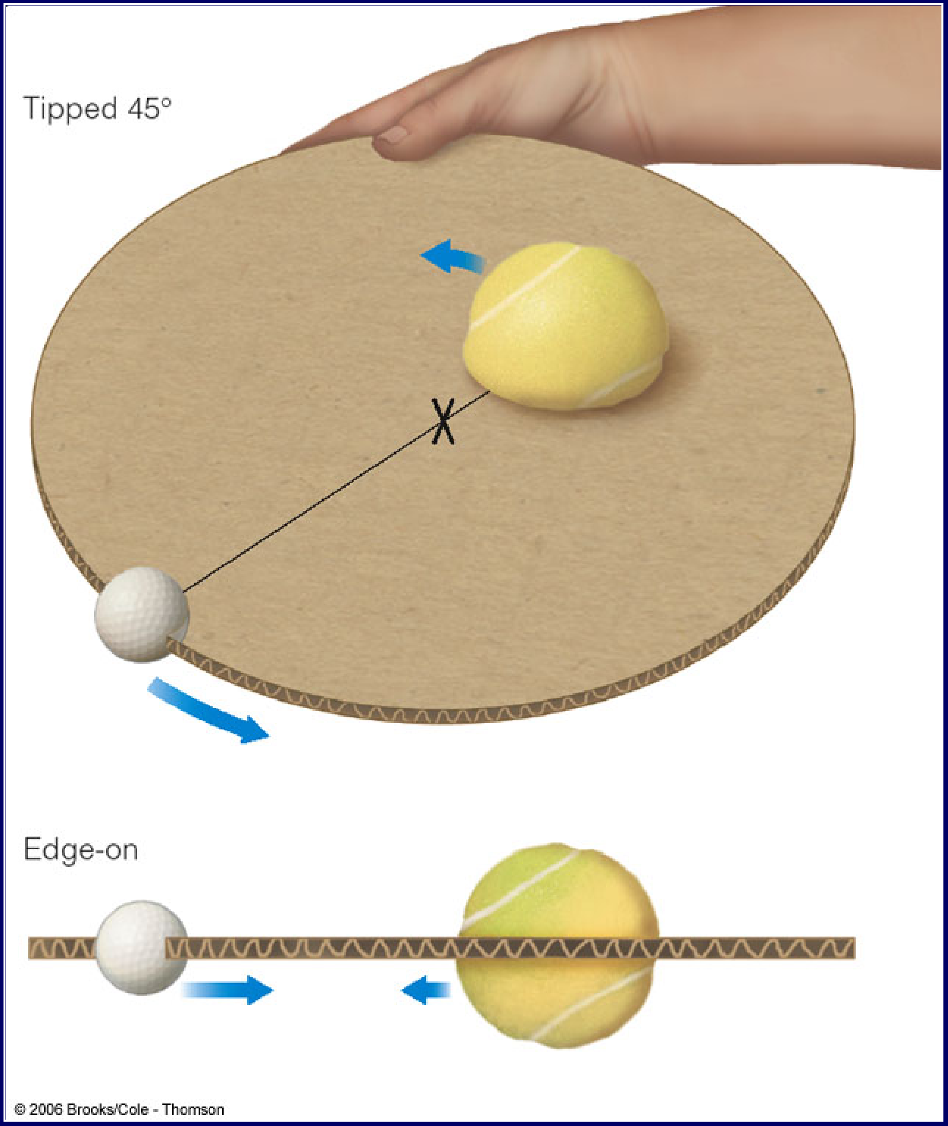
Eclipsing Binaries

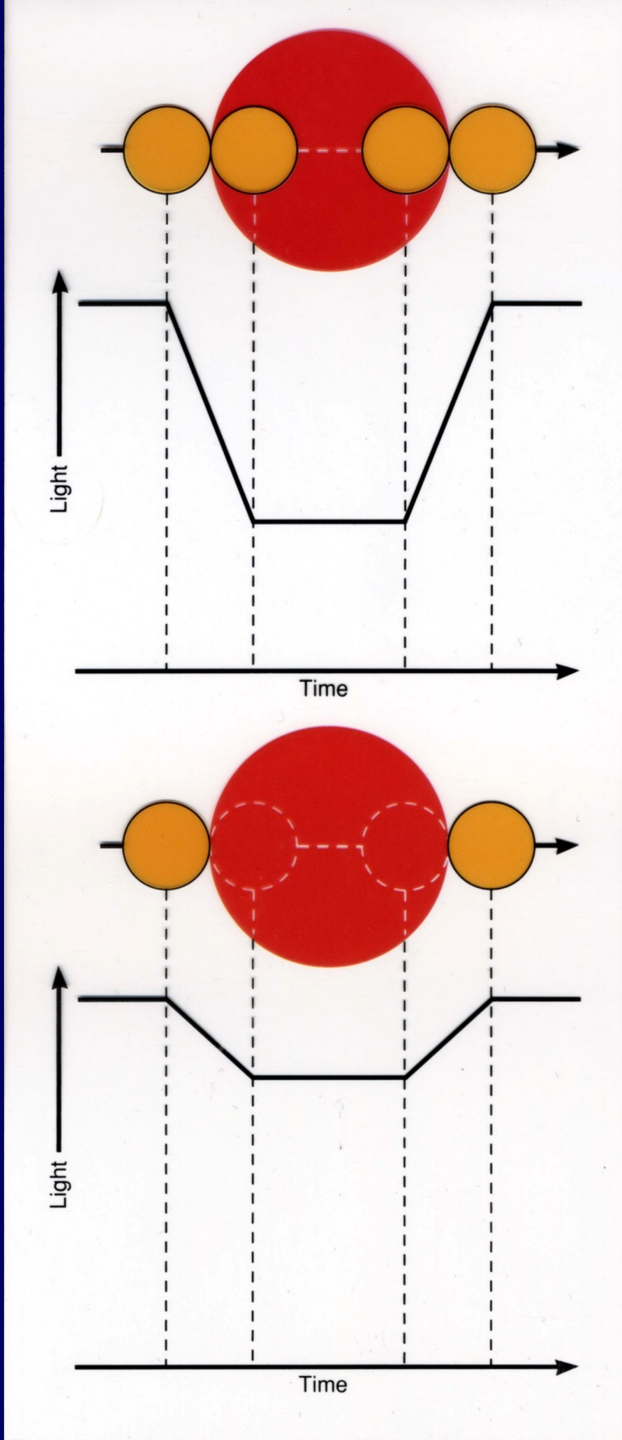
Usually, the inclination angle of a binary system is unknown $\Rightarrow$ uncertainty in mass estimates.

Special case:

Eclipsing Binaries

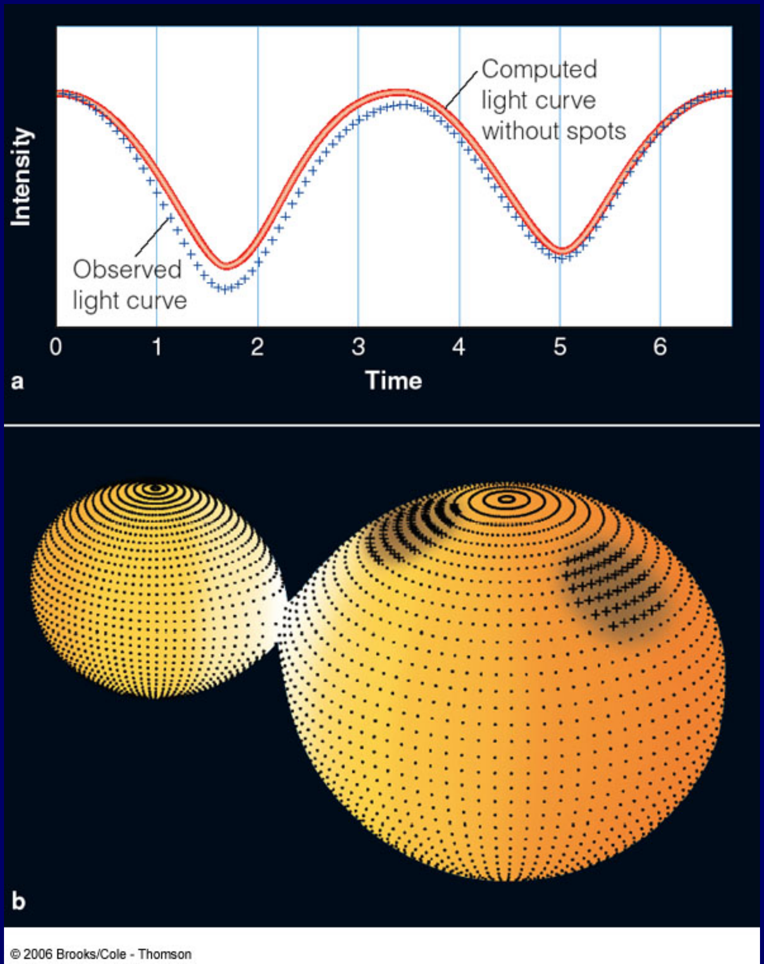
Here, we know that we are looking at the system edge-on (~0° inclination)!





Eclipsing Binaries

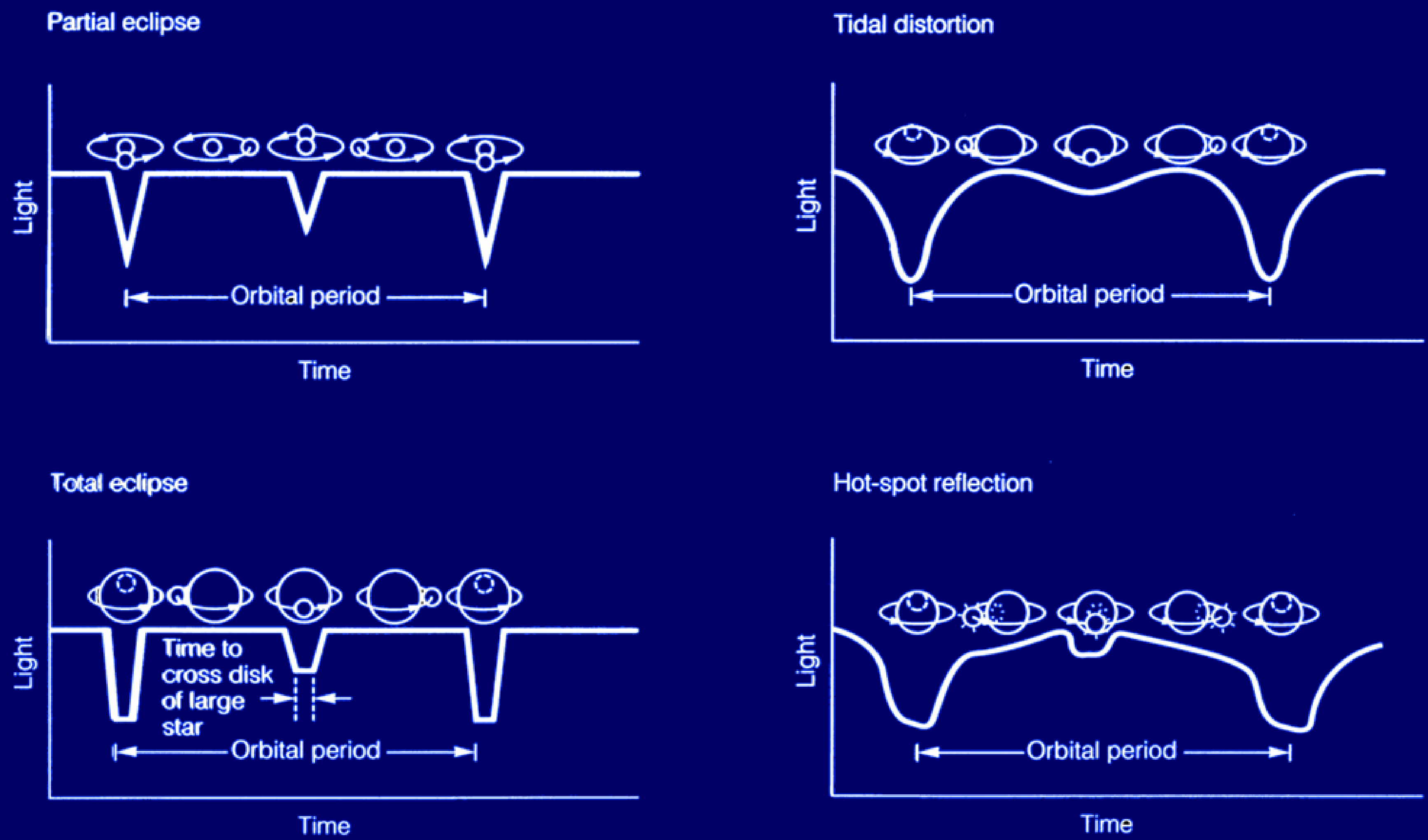
Produce a "double-dip" light curve



Example: VW Cephei

Eclipsing Binaries

Unfortunately, light curves are not always easy to interpret

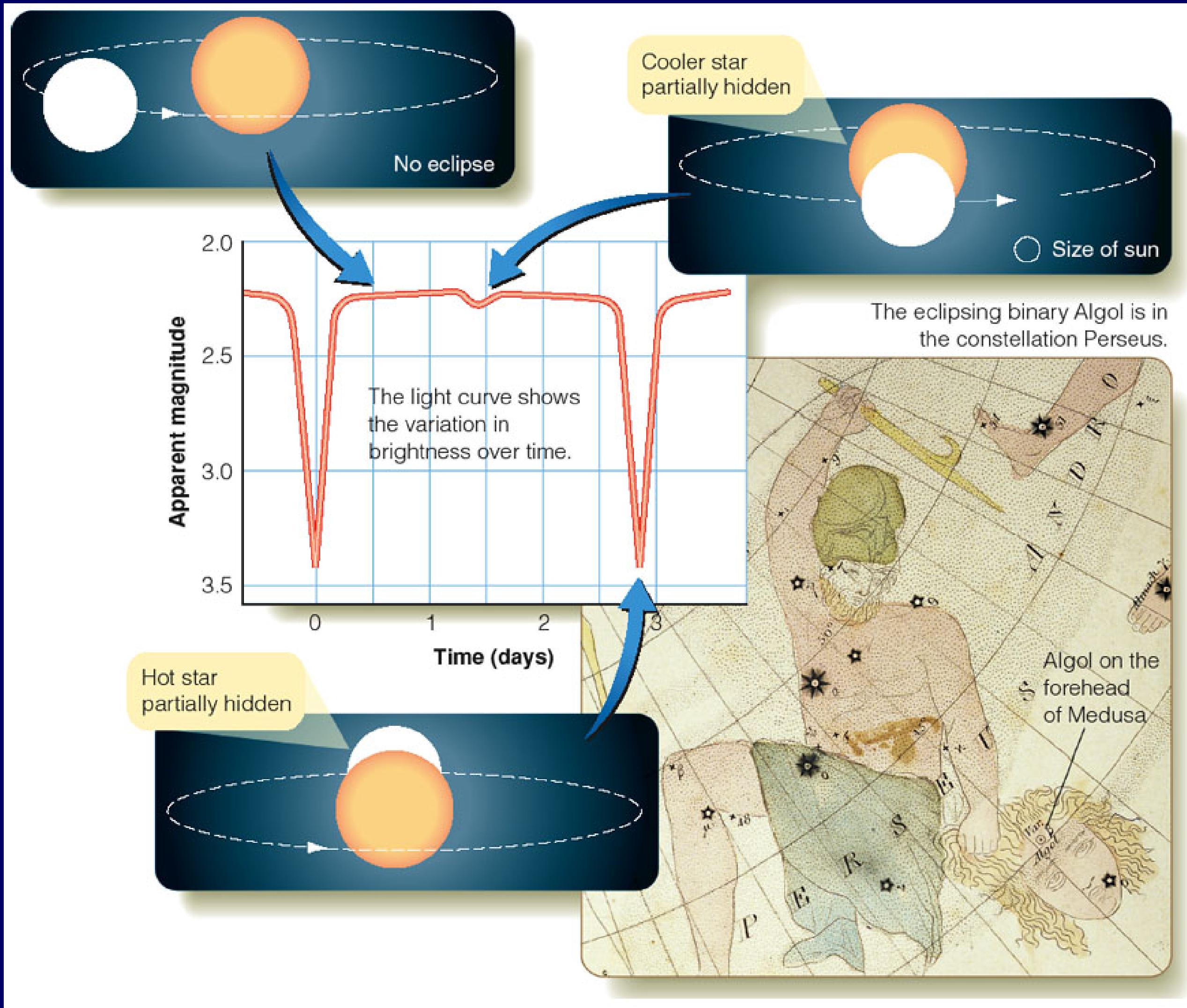


Eclipsing Binaries

Example:

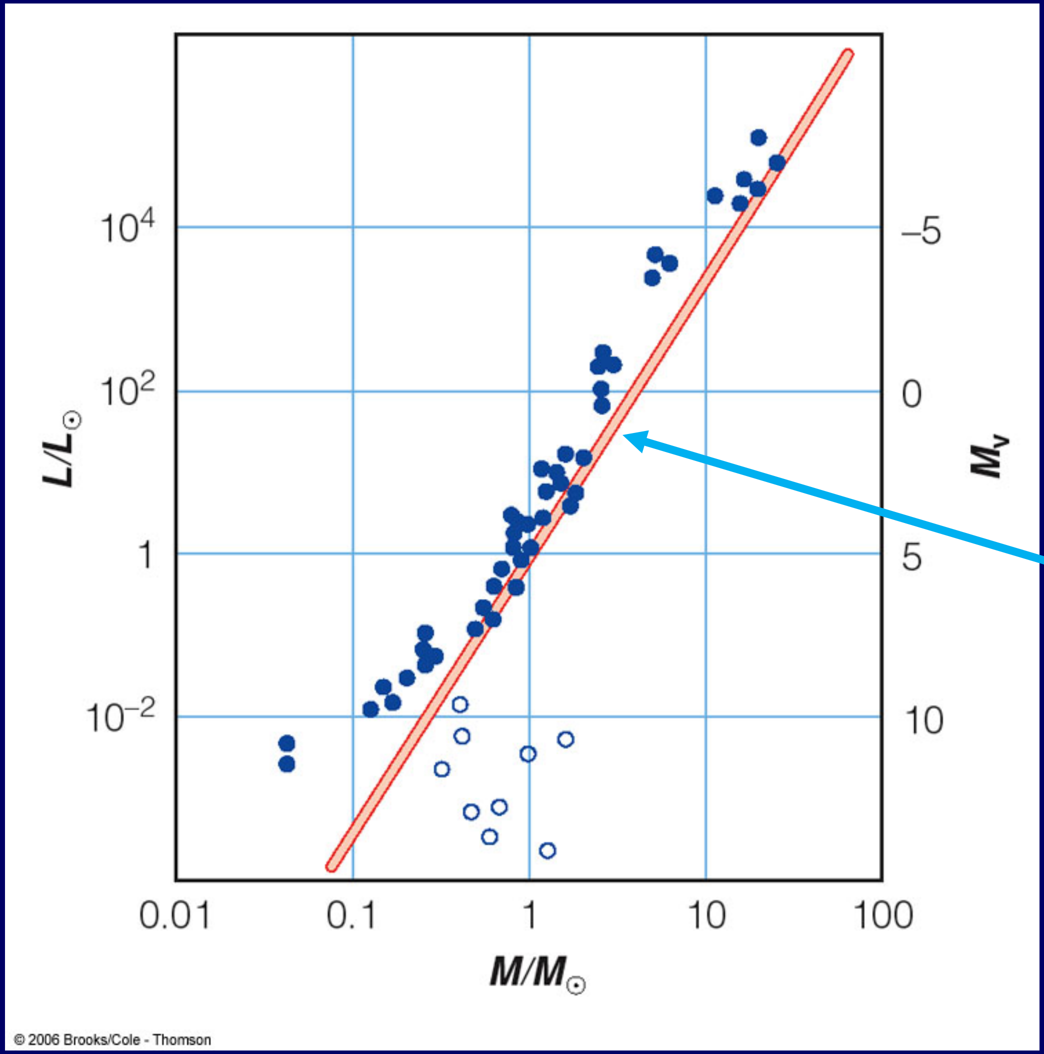
Algol in the constellation Perseus

From the light curve of Algol, we can infer that the system contains two stars of very different surface temperatures orbiting in a slightly inclined plane.



The Mass-Luminosity Relation

Studying binaries enables us to obtain the masses of some stars. Plotting luminosity vs. mass shows that they are related.



More massive stars are more luminous.

$$\boxed{L \propto M^{3.5}}$$

Masses of Stars in the H-R Diagram

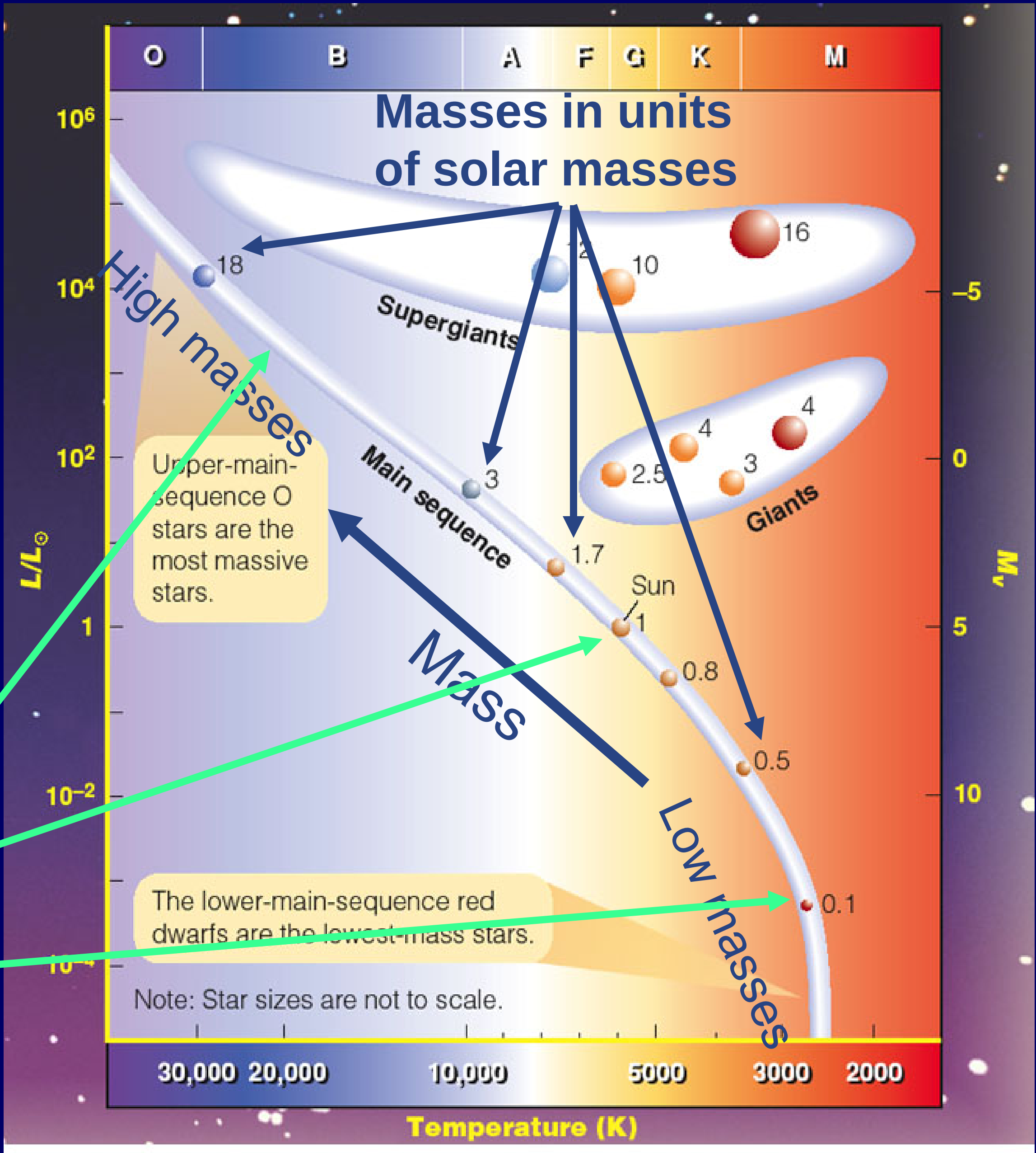
We will learn that high-mass stars have much shorter lives than low-mass stars:

$$t_{life} \sim M^{-2.5}$$

10 $m_{\odot}$: ~30 million yr.

Sun: ~10 billion yr.

0.1 $m_{\odot}$: ~3 trillion yr.



A Census of the Stars

Faint, red dwarfs (low mass) are the most common stars.

Bright, hot, blue main-sequence stars (high-mass) are rare.

Giants and supergiants are extremely rare.