7.66. A truck with mass $m$ has a brake failure while going down an icy mountain road of constant downward slope angle $\alpha$ (see figure). Initially the truck is moving downhill at speed $v_0$. After careening downhill a distance $L$ with negligible friction, the truck driver steers the runaway vehicle onto a runaway truck ramp of constant upward slope angle $\beta$. The truck ramp has a soft sand surface for which the coefficient of rolling friction is $\mu_r$. What is the distance that the truck moves up the ramp before coming to a halt? Solve using energy methods.

Identify: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the initial and final positions of the truck.

Set up: Let $y = 0$ at the lowest point of the path of the truck. $W_{\text{other}}$ is the work done by friction. $f_r = \mu_r n = \mu_r mg \cos \beta$.

Execute: Denote the distance the truck moves up the ramp by $x$. $K_1 = \frac{1}{2}mv_0^2$, $U_1 = mgL \sin \alpha$, $K_2 = 0$, $U_2 = mgx \sin \beta$ and $W_{\text{other}} = -\mu_r mgx \cos \beta$. From $W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1)$, and solving for $x$,

$$x = \frac{K_1 + mgL \sin \alpha}{mg(\sin \beta + \mu_r \cos \beta)} = \frac{(v_0^2/2g) + L \sin \alpha}{\sin \beta + \mu_r \cos \beta}.$$

Evaluate: $x$ increases when $v_0$ increases and decreases when $\mu_r$ increases.