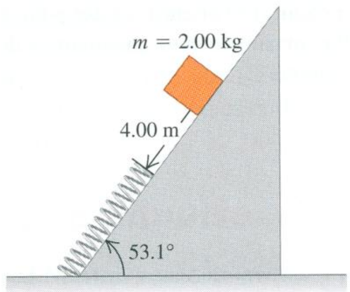
7.74. A 2.00-kg package is released on a 53.1° incline 4.00 m from a long spring with force constant 120 N/m that is attached at the bottom of the incline (see figure) The coefficients of friction between the package and the incline are μ_s = 0.04 and μ_k = 0.20. The mass of the spring is negligible. (a) What is the speed of the package just before it reaches the spring? (b) What is the maximum compression of the spring? (c) The package rebounds back up the incline. How close does it get to its initial position?



Identify: Apply Eq.(7.14) to the motion of the package. $W_{\text{other}} = W_{f_k}$, the work done by the kinetic friction force.

Set up: $f_k = \mu_k n = \mu_k mg\cos\theta$, with $\theta = 53.1°$. Let $L = 4.00\text{ m}$, the distance the package moves before reaching the spring and let *d* be the maximum compression of the spring. Let point 1 be the initial position of the package, point 2 be just as it contacts the spring, point 3 be at the maximum compression of the spring, and point 4 be the final position of the package after it rebounds.

Execute:

(a) $K_1 = 0$, $U_2 = 0$, $W_{\text{other}} = -f_k L = -\mu_k L\cos\theta$. $U_1 = mgL\sin\theta$. $K_2 = \frac{1}{2}mv^2$, where *v* is the speed before the block hits the spring. Eq.(7.14) applied to points 1 and 2, with $y_2 = 0$, gives $U_1 + W_{\text{other}} = K_2$. Solving for *v*,

$$v = \sqrt{2gL(\sin\theta - \mu_k\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(4.00\text{ m})(\sin 53.1° - (0.20)\cos 53.1°)} = 7.30\text{ m/s}.$$

(b) Apply Eq.(7.14) to points 1 and 3. Let $y_3 = 0$. $K_1 = K_3 = 0$. $U_1 = mg(L+d)\sin\theta$. $U_2 = \frac{1}{2}kd^2$. $W_{\text{other}} = -f_k(L+d)$. Eq.(7.14) gives $mg(L+d)\sin\theta - \mu_k mg\cos\theta(L+d) = \frac{1}{2}kd^2$. This can be written as $d^2\dfrac{k}{2mg(\sin\theta - \mu_k\cos\theta)} - d - L = 0.$ The factor multiplying d^2 is 4.504 m^{-1}, and use of the quadratic formula gives $d = 1.06\text{ m}$.

(c) The easy thing to do here is to recognize that the presence of the spring determines *d*, but at the end of the motion the spring has no potential energy, and the distance below the starting point is determined solely by how much energy has been lost to friction. If the block ends up a distance *y* below the starting point, then the block has moved a distance $L+d$ down the incline and $L+d-y$ up the incline. The magnitude of the friction force is the same in both directions, $\mu_k mg\cos\theta$, and so the work done by friction is $-\mu_k(2L+2d-y)mg\cos\theta$. This must be equal to the change in gravitational potential energy, which is $-mgy\sin\theta$. Equating these and solving for *y* gives

$y = (L+d)\dfrac{2\mu_k \cos\theta}{\sin\theta + \mu_k \cos\theta} = (L+d)\dfrac{2\mu_k}{\tan\theta + \mu_k}$. Using the value of *d* found in part (b) and the given values for μ_k and θ gives $y = 1.32\text{ m}$.

Evaluate: Our expression for *y* gives the reasonable results that $y = 0$ when $\mu_k = 0$; in the absence of friction the package returns to its starting point.