Supplementary Exercises and Exam Questions
Discrete Mathematics with Applications, 3rd Edition
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Chapter 5

1. Let A and B be sets. Define precisely (but concisely) what it means for A to be a subset of B.

2. Write a negation for the following statement:

$$\text{For all } x, \text{ if } x \in A \cap B \text{ then } x \in B.$$

3. Fill in the blanks in the following sentence: If A, B and C are any sets, then by definition of set difference $x \in A - (B \cap C)$ if, and only if, x ______ and x ______.

4.
   (a) Is $2 \subseteq \{2, 4, 6\}$?
   (b) Is $\{3\} \in \{1, 3, 5\}$?

5. If $X = \{u, v\}$, what is the power set of X?

6. Fill in the blanks:
   (a) Given sets A and B, to prove that $(A - B) \cup (A \cap B) \subseteq A$, we suppose that $x \in$ ______ and we must show that $x \in$ ______.
   (b) By definition of union, to say that $x \in (A - B) \cup (A \cap B)$ means that ______.

7. Define sets A and B as follows: $A = \{n \in \mathbf{Z} \mid n = 8r - 3 \text{ for some integer } r\}$ and $B = \{m \in \mathbf{Z} \mid m = 4s + 1 \text{ for some integer } s\}$.
   (a) Is $A \subseteq B$?
   (b) Is $B \subseteq A$?

   Justify your answers carefully. (In other words, provide a proof if the statement is true or a disproof if the statement is false.)

8. Let $X = \{l \in \mathrm{Z} \mid l = 5a + 2 \text{ for some integer } a\}$, $Y = \{m \in \mathrm{Z} \mid m = 4b + 3 \text{ for some integer } b\}$, and $Z = \{n \in \mathrm{Z} \mid n = 4c - 1 \text{ for some integer } c\}$.
   (a) Is $X \subseteq Y$?
   (b) Is $Y \subseteq Z$?

   Justify your answers carefully. (In other words, provide a proof if the statement is true or a disproof if the statement is false.)

9. The following is an outline of a proof that $(A \cup B)^c \subseteq A^c \cap B^c$. Fill in the blanks.

   Proof: Given sets A and B, to prove that $(A \cup B)^c \subseteq A^c \cap B^c$, we suppose $x \in$ __(a)__ and then we show that $x \in$ __(b)__. So suppose that __(c)__. Then by definition of complement, __(d)__. So by definition of union, it is not the case that (x is in A or x is in B). Consequently, x is not in A __(e)__ x is not in B because of De Morgan's law of logic. In symbols, this says that $x \notin A$ and $x \notin B$. So by definition of complement, $x \in$ __(f)__ and $x \in$ __(g)__. Thus, by definition of intersection, $x \in$ __(h)__. *[as was to be shown]*.

10. Prove the following statement using an element argument and reasoning directly from the definitions of union, intersection, set difference.

$$\text{For all sets } A,\ B, \text{ and } C,\ (A \cup B) \cap C \subseteq A \cup (B \cap C).$$

11. Disprove the following statement by finding a counterexample.

$$\text{For all sets } A,\ B, \text{ and } C,\ A \cup (B \cap C) \subseteq (A \cup B) \cap C.$$

12. Consider the statement

$$\text{For all sets } A \text{ and } B,\ (A - B) \cap B = \emptyset.$$

The proof below is the beginning of a proof using the element method for prove that a set equals the empty set. Complete the proof without using any of the set properties from Theorem 5.2.2.

Proof: Suppose the given statement is false. Then there exist sets A and B such that $(A - B) \cap B \neq \emptyset$. Thus there is an element x in $(A - B) \cap B$. By definition of intersection,. . . .

13. Consider the statement

$$\text{For all sets } A \text{ and } B,\ (A - B) \cap B = \emptyset.$$

Complete the proof begun below in which the given statement is derived algebraically from the properties on the attached sheet. Be sure to give a reason for every step that exactly justifies what was done in the step:

Proof:

Let A and B be any sets. Then the left-hand side of the equation to be shown is

$$\begin{aligned}
(A - B) \cap B &= (A \cap B^c) \cap B && \text{by the ______ law} \\
&= \text{______} && \text{by the ______ law} \\
&= \text{______} && \text{by the ______ law} \\
&= \text{______} && \text{by the ______ law} \\
&= \text{______} && \text{by the ______ law}
\end{aligned}$$

which is the right-hand side of the equation to be shown. *[Hence the given statement is true.]*

(The number of lines in the outline shown above are just meant to be suggestive. To complete the proof you may need more lines or you may be able to do it with fewer lines. Use however many lines as you need.)

14.

(a) Prove the following statement using the element method for prove that a set equals the empty set: For all sets A and B, $A \cap (B - A) = \emptyset$.

(b) Use the properties in Theorem 5.2.2 to prove the statement in part (a). Be sure to give a reason for every step.

15. Derive the following result "algebraically" using the properties listed in Theorem 5.2.2 (and reproduced on the attached sheet). Give a reason for every step.

$$\text{For all sets } A,\ B, \text{ and } C,\ (A \cup C) - B = (A - B) \cup (C - B).$$

16. Derive the following result. You may do so either "algebraically" using the properties listed in Theorem 5.2.2, being sure to give a reason for every step, or you may use the element method for proving a set equals the empty set.

$$\text{For all sets } B \text{ and } C,\ (B - C) - B = \emptyset.$$

17. Use the element method for proving a set equals the empty set to prove that

$$\text{For all sets } A \text{ and } C,\ (A - C) \cap (C - A) = \emptyset.$$

18. Is the following sentence a statement: This sentence is false or $-2^2 = 4$. Justify your answer.