**Supplementary Exercises and Exam Questions**
**Discrete Mathematics with Applications, 3rd Edition**
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# Chapter 6

1. On each of three consecutive days the National Weather Service announces that there is a 50-50 chance of rain. Assuming that the National Weather Service is correct, what is the probability that it rains on at most one of the three days? Justify your answer. (*Hint*: Represent the outcome that it rains on day 1 and doesn't rain on days 2 and 3 as RNN.)

2. How many elements are in the one-dimensional array shown below?

$$A[7],\ A[8],\ \ldots\ ,\ A\left[\left\lfloor \frac{145}{2} \right\rfloor\right]$$

3. In a certain state, license plates each consist of 2 letters followed by 3 digits.
   (a) How many different license plates are there?
   (b) How many different license plates are there that have no repeated letters or digits?

4. In a certain state, license plates each consist of 2 letters followed by either 3 or 4 digits. How many different license plates are there that have no repeated letters or digits?

5. Suppose there are three routes from Byrne Hall to McGaw Hall and five routes from McGaw Hall to Monroe Hall. How many ways is it possible to travel from Byrne Hall to Monroe Hall by way of McGaw Hall?

6. In a certain discrete math class, three quizzes were given. Out of the 30 students in the class:

   15 scored 12 or above on quiz #1,
   12 scored 12 or above on quiz #2,
   18 scored 12 or above on quiz #3,
   7 scored 12 or above on quizzes #1 and #2,
   11 scored 12 or above on quizzes #1 and #3,
   8 scored 12 or above on quizzes #2 and #3,
   4 scored 12 or above on quizzes #1, #2, and #3.

   (a) How many scored 12 or above on at least one quiz?
   (b) How many scored 12 or above on quizzes 1 and 2 but not 3?

7. A club has seven members. Three are to be chosen to go as a group to a national meeting.
   (a) How many distinct groups of three can be chosen?
   (b) If the club contains four men and three women, how many distinct groups of three contain two men and one woman?
   (c) If the club contains four men and three women, how many distinct groups of three contain at most two men?
   (d) If the club contains four men and three women, how many distinct groups of three contain at least one woman?
   (e) If the club contains four men and three women, what is the probability that a distinct group of three will contain at least one woman?

(f) If two members of the club refuse to travel together as part of the group (but each is willing to go if the other does not), how many distinct groups of three can be chosen?

(g) If two members of the club insists on either traveling together or not going at all, How many distinct groups of three can be chosen?

8. Suppose that a fair coin is tossed ten times.

   (a) How many ways can at least eight heads be obtained?

   (b) What is the probability of obtaining at least eight heads?

9. A large pile of coins consists of pennies, nickels, dimes, and quarters (at least 20 of each).

   (a) How many different collections of 20 coins can be chosen?

   (b) How many different collections of 20 coins chosen at random will contain at least 3 coins of each type?

   (c) What is the probability that a collection of 20 coins chosen at random will contain at least 3 coins of each type?

10. Prove for all integers $n$, $k$, and $r$ with $n \geq k \geq r$ that $\binom{n}{k}\binom{k}{r} = \binom{n}{r}\binom{n-r}{k-r}$.

11. The binomial theorem states that for any real numbers $a$ and $b$,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \qquad \text{for any integer } n \geq 0.$$

Use this theorem to compute $(2x-1)^5$.

12. The binomial theorem states that for any real numbers $a$ and $b$,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \qquad \text{for any integer } n \geq 0.$$

Use this theorem to show that for any integer $n \geq 0$, $\sum_{k=0}^{n} (-1)^k \binom{n}{k} 3^{n-k} 2^k = 1$.

13. Express the following sum in closed form (without using a summation symbol and without using an ellipsis $\ldots$): $\sum_{k=0}^{n} \binom{n}{k} 7^k$.

14. Let $A$, $B$, and $C$ be events in a sample space $S$ such that $S = A \cup B \cup C$. Suppose that $P(A) = 0.3$, $P(B) = 0.6$, and $P(A \cap B) = 0.2$. Find each of the following.

   (a) $P(A \cup B)$

   (b) $P(C)$

   (c) $P(A^c \cup B^c)$

15. An urn contains four balls numbered 1, 3, 4, and 6. If a person selects a set of two balls at random, what is the expected value of the product of the numbers on the balls?

16. Suppose $A$ and $B$ are events in a sample space $S$, and $P(A|B) = 1/2$ and $P(B) = 1/3$. What is $P(A \cap B)$?

17. A teacher offers ten possible assignments for extra credit in a course but requires students to choose them, without looking, from a hat. Six assignments involve library research and four are computer programming exercises. Suppose that a student chooses two assignments, one after the other, at random without replacement.

(a) What is the probability that both assignments are computer programming exercises?

(b) What is the probability that at least one of the assignments is a computer programming exercise?

18. A screening test for a certain disease is used in a large population of people of whom 1 in 1000 actually have the disease. Suppose that the false positive rate is 1% and the false negative rate is 0.5%. Thus a person who has the disease tests positive for it 99.5% of the time, and a person who does not have the disease tests negative for it 99% of the time.

    (a) What is the probability that a randomly chosen person who tests positive for the disease actually has the disease?

    (b) What is the probability that a randomly chosen person who tests negative for the disease actually has the disease?

19. A coin is loaded so that the probability of heads is 0.55 and the probability of tails is 0.45. Suppose the coin is tossed twice and the results of the tosses are independent.

    (a) What is the probability of obtaining exactly two heads?

    (b) What is the probability of obtaining exactly one head?

    (c) What is the probability of obtaining no heads?

    (d) What is the probability of obtaining at least one head?